Appendix

Section A provides a proof that isometry preserves angles. Section B derives the closed-form of the gradient projection on the tangent space at a point in the Stiefel manifold. Section C gives further experimental results. Section D lists the grid considered for hyper-parameters.

A Isometry Preserves Angles

Theorem A.1. *T is an isometry iff it preserves inner products.*

Proof. Suppose T is an isometry. Then for any $v, w \in V$,

$$\|T(v) - T(w)\|^2 = \|v - w\|^2$$
$$\langle T(v) - T(w), T(v) - T(w)\rangle = \langle v - w, v - w\rangle$$
$$\|T(v)\|^2 + \|T(w)\|^2 - 2\langle T(v), T(w)\rangle = \|v\|^2 + \|w\|^2 - 2\langle v, w\rangle.$$

Since $\|T(u)\| = \|u\|$ for any u in V, all the length squared terms in the last expression above cancel out and we get

$$\langle T(v), T(w)\rangle = \langle v, w\rangle.$$

Conversely, if T preserves inner products, then

$$\langle T(v - w), T(v - w)\rangle = \langle v - w, v - w\rangle,$$

which implies

$$\|T(v - w)\| = \|v - w\|,$$

and since T is linear,

$$\|T(v) - T(w)\| = \|v - w\|.$$

This shows that T preserves distance. □

B Closed-form of Projection in Tangent Space

This section closely follows the arguments of Tagare [2011].

Let $\{X \in \mathbb{R}^{n\times p} | X^\top X = I\}$ defines a manifold in Euclidean space $\mathbb{R}^{n\times p}$, where $n > p$. This manifold is called the Stiefel manifold. Let $\mathcal{T}_X$ denotes a tangent space at X.

Lemma B.1. *Any $Z \in \mathcal{T}_X$ satisfies:*

$$Z^\top X + X^\top Z = 0$$

i.e. *$Z^\top X$ is a skew-symmetric $p \times p$ matrix.*

Note, that X consists of p orthonormal vectors in $\mathbb{R}^n$. Let $X_\perp$ be a matrix consisting of the additional $n - p$ orthonormal vectors in $\mathbb{R}^n$ *i.e.* $X_\perp$ lies in the orthogonal compliment of X, $X^\top X_\perp = 0$. The concatenation of X and $X_\perp$, $[XX_\perp]$ is $n \times n$ orthonormal matrix. Then, any matrix $U \in \mathbb{R}^{n\times p}$ can be represented as: $U = XA + X_\perp B$, where A is a $p \times p$ matrix, and B is a $(n - p) \times p$ matrix.

Lemma B.2. *A matrix $Z = XA + X_\perp B$ belongs to the tangent space at a point on Stiefel manifold $\mathcal{T}_X$ iff A is skew-symmetric.*

Let $G \in \mathbb{R}^{n\times p}$ be the gradient computed at X. Let the projection of the gradient on the tangent space is denoted by $\pi_{\mathcal{T}_X}(G)$.

Lemma B.3. *Under the canonical inner product, the projection of the gradient on the tangent space is given by $\pi_{\mathcal{T}_X}(G) = AX$, where $A = GX^\top - XG^\top$.*

Proof. Express $G = XG_A + X_\perp G_B$. Let Z be any vector in the tangent space, expressed as $Z = XZ_A + X_\perp Z_B$, where Z_A is a skew-symmetric matrix according to B.2. Therefore,

$$\begin{aligned} \pi_{\mathcal{T}_X}(G) &= \text{tr}(G^\top Z), \\ &= \text{tr}((XG_A + X_\perp G_B)^\top (XZ_A + X_\perp Z_B)), \\ &= \text{tr}(G_A^\top Z_A + G_B^\top Z_B). \end{aligned} \tag{11}$$

Writing G_A as $G_A = \text{sym}(G_A) + \text{skew}(G_A)$, and plugging in (11) gives,

$$\pi_{\mathcal{T}_X}(G) = \text{tr}(\text{skew}(G_A)^\top Z_A + G_B^\top Z_B). \tag{12}$$

Let $U = XA + X_\perp B$ is the vector that represents the projection of G on the tangent space at X. Then,

$$\begin{aligned} \langle U, Z\rangle_c &= \text{tr}(U^\top (I - \frac{1}{2}XX^\top)Z), \\ &= \text{tr}((XA + X_\perp B)^\top (I - \frac{1}{2}XX^\top)(XZ_A + X_\perp Z_B)), \\ &= \text{tr}(\frac{1}{2}A^\top Z_A + B^\top Z_B) \end{aligned} \tag{13}$$

By comparing (12) and (13), we get $A = 2\text{skew}(G_A)$ and $B = G_B$. Thus,

$$\begin{aligned} U &= 2X\text{skew}(G_A) + X_\perp G_B, \\ &= X(G_A - G_A^\top) + X_\perp G_B, \quad \because \text{skew}(G_A) = \frac{1}{2}(G_A - G_A^\top) \\ &= XG_A - XG_A^\top + G - XG_A, \quad \because G = XG_A + X_\perp G_B \\ &= G - XG_A^\top, \\ &= G - XG^\top X, \quad \because G_A = X^\top G, \\ &= GX^\top X - XG^\top X, \\ &= (GX^\top - XG^\top)X \end{aligned}$$

□

C More Results

Table 3: *Accuracy* (2) *and Forgetting* (3) *results of continual learning experiments for larger episodic memory sizes.* 2, 3 *and* 5 *samples per class per task are stored, respectively. Top table is for Split CIFAR. Bottom table is for Split miniImageNet.*

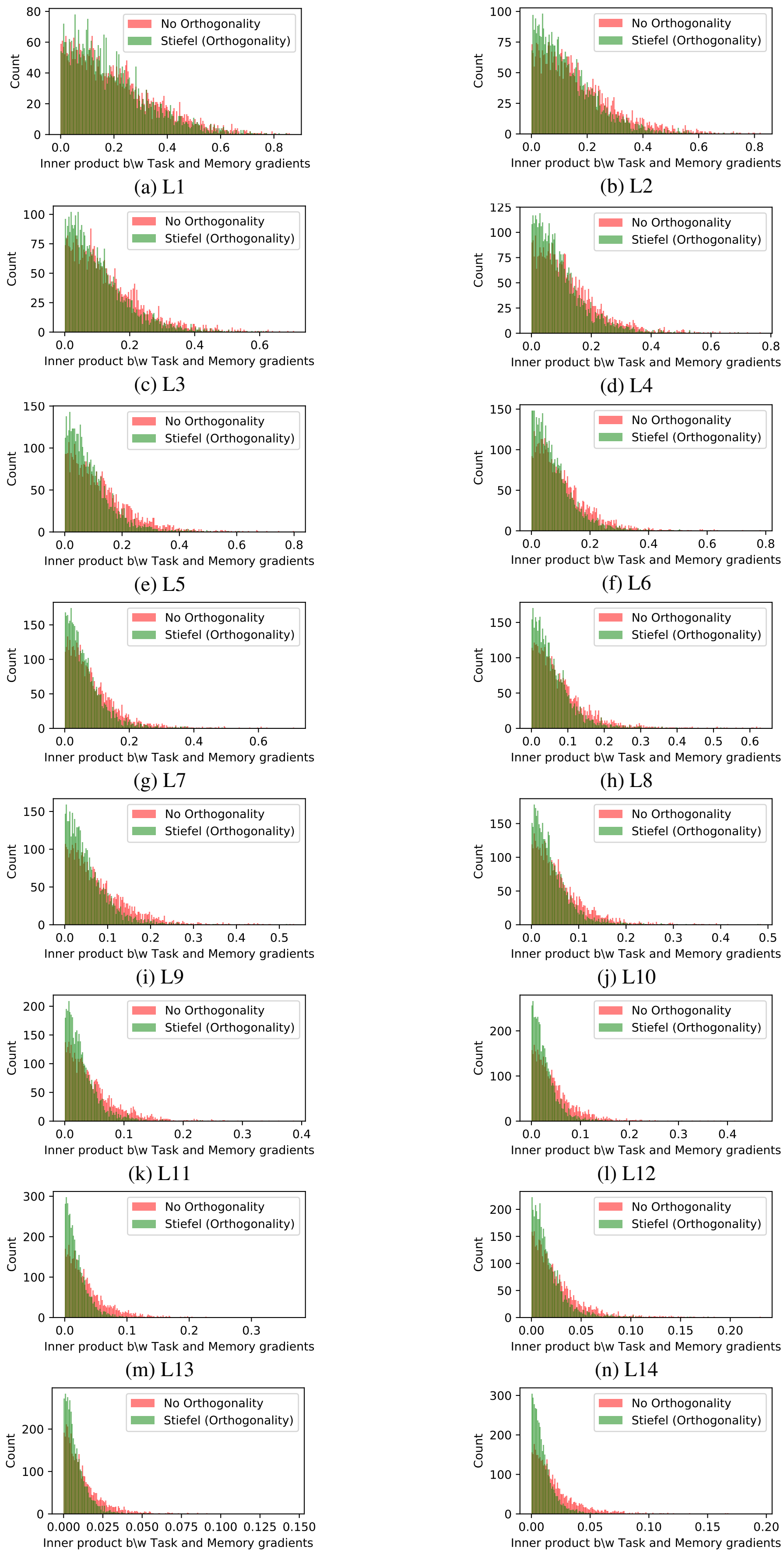
METHOD	ACCURACY			FORGETTING		
	2	3	5	2	3	5
AGEM	52.2 (±2.59)	56.1 (±1.52)	60.9 (±2.50)	0.16 (±0.01)	0.13 (±0.01)	0.11 (±0.01)
ER-RING	61.9 (±1.92)	64.8 (±0.77)	67.2 (±1.72)	0.11 (±0.02)	0.08 (±0.01)	0.06 (±0.01)
ORTHOG-SUBSPACE	64.7 (±0.53)	66.8 (±0.83)	67.3 (±0.98)	0.07 (±0.01)	0.05 (±0.01)	0.05 (±0.01)

METHOD	ACCURACY			FORGETTING		
	2	3	5	2	3	5
AGEM	45.2 (±2.35)	47.5 (±2.59)	49.2 (±3.35)	0.14 (±0.01)	0.13 (±0.01)	0.10 (±0.01)
ER-RING	51.2 (±1.99)	53.9 (±2.04)	56.8 (±2.31)	0.10 (±0.01)	0.09 (±0.02)	0.06 (±0.01)
ORTHOG-SUBSPACE	53.4 (±1.23)	55.6 (±0.55)	58.2 (±1.08)	0.07 (±0.01)	0.06 (±0.01)	0.05 (±0.01)

D Hyper-parameter Selection

In this section, we report the hyper-parameters grid considered for experiments. The best values for different benchmarks are given in parenthesis.

- Multitask
  - `learning rate: [0.003, 0.01, 0.03 (CIFAR, miniImageNet), 0.1 (MNIST perm, rot), 0.3, 1.0]`
- Finetune
  - `learning rate: [0.003, 0.01, 0.03 (CIFAR, miniImageNet), 0.1 (MNIST perm, rot), 0.3, 1.0]`
- EWC
  - `learning rate: [0.003, 0.01, 0.03 (CIFAR, miniImageNet), 0.1 (MNIST perm, rot), 0.3, 1.0]`
  - `regularization: [0.1, 1, 10 (MNIST perm, rot, CIFAR, miniImageNet), 100, 1000]`
- AGEM
  - `learning rate: [0.003, 0.01, 0.03 (CIFAR, miniImageNet), 0.1 (MNIST perm, rot), 0.3, 1.0]`
- MER
  - `learning rate: [0.003, 0.01, 0.03 (MNIST, CIFAR, miniImageNet), 0.1, 0.3, 1.0]`
  - `within batch meta-learning rate: [0.01, 0.03, 0.1 (MNIST, CIFAR, miniImageNet), 0.3, 1.0]`
  - `current batch learning rate multiplier: [1, 2, 5 (CIFAR, miniImageNet), 10 (MNIST)]`
- ER-Ring
  - `learning rate: [0.003, 0.01, 0.03 (CIFAR, miniImageNet), 0.1 (MNIST perm, rot), 0.3, 1.0]`
- ORTHOG-SUBSPACE
  - `learning rate: [0.003, 0.01, 0.03, 0.1 (MNIST perm, rot), 0.2 (miniImageNET), 0.4 (CIFAR), 1.0]`



(a) L1 (b) L2 (c) L3 (d) L4 (e) L5 (f) L6 (g) L7 (h) L8 (i) L9 (j) L10 (k) L11 (l) L12 (m) L13 (n) L14 (o) L15 (p) L16

Figure 3: *Histogram of inner product of current task and memory gradients in all layers in Split CIFAR.*