

1 **Thank you reviewers.**

2 **R1:** «*Setting Questions*»: The task sequence is arbitrary and is chosen by the *environment*, which may be adversarial
3 (lines 83-86). Thus the environment might present the instance sequence for different tasks in a given order. Alternatively
4 the active task on trial t could be determined by the environment at random, etc. You ask “... *what is the difference. ...*
5 *switching constraints ... memory*”: The bound is completely independent of the task sequence.

6 **R2:** “...*algorithms require some prior knowledge ...*”, “*parameter-free design*” : See lines 293-295. Automatic
7 parameter tuning has been considered since the advent of online learning, using e.g. “doubling trick”, “adaptive
8 parameter rates” and “mixtures over parameters”; there is no reason to suppose such methods would not apply here.

9 “*further survey .. contextual bandit ..*” : Thank you for the references. We will include them in the manuscript. Note,
10 however, **none of them consider long term memory, the key theoretical concept of this paper.**

11 “*Consider multitask... compare their regret upper bounds..individually*” : These comparisons follow directly from the
12 theorem statements. E.g., in Thm. 3, we would pay the learner complexity term $\sum_{h \in m(h^*)} \|h\|_K^2 X_K^2$ on a per-task
13 basis. Finally this gain in the multitask case is not just in the upper-bound but it is also reflected by the lower-bound
14 (see Prop. 4).

15 **R3:** Thank you for your comments.

16 **R4:** “*In the online multitask expert setting, I would suggest providing a more rigorous comparison with the related*
17 *work. As indicated above, it seems that the present framework can be cast as a problem already investigated in [1]*” :
18 We provide a rigorous comparison. See lines 167-169 for a discussion that includes [1] where we say about their model
19 “a special case of ours where each task is associated only with a single hypothesis, i.e., no internal switching within a
20 task.” Your proposed reduction would not even enable vanilla switching in a single task setting! Unless the environment
21 sent a signal to switch comparators, which trivializes switching altogether.

22 Further, comparing the framework of [1] to this paper. In [1] they are interested in a model where each task is associated
23 with single comparator, i.e., each task in our Figure 1 would have a *single* color, **not** a sequence of colors. I suspect that
24 your confusion comes from a misinterpretation of [1, Corollary 1]. When the authors of [1] use the word “switch” they
25 do not mean a switch in the “comparator sequence,” but a switch in the “task-query sequence” (our vector ℓ) i.e, if the
26 environment queries task 1 on trial 1 and then queries task 5 on trial 2, then their regret bound pays a $\ln m$ for each such
27 “switch.” For our bound such switches of task vector incur NO increase in regret, i.e., we strictly improve.

28 “*In the online multitask linear setting - and more generally the RKHS setting (Section 4), there is*
29 *something wrong about the results. ... In this setting, it is well-known that for linear functions of*
30 *dimension $d \geq 2$ the Littlestone dimension is infinite (see e.g. [52]). So, we cannot hope to find*
31 *sublinear regret bounds for linear functions with respect to the zero-one loss!*”

32 It seems that the source of your confusion is that you did not understand what is an RKHS \mathcal{H}_K or the definition of
33 $\mathcal{H}_K^{(x)}$ (recalling lines 58-61).

34 Given a reproducing kernel $K : \mathcal{X} \times \mathcal{X} \rightarrow \mathfrak{R}$ we denote the induced norm of the reproducing kernel
35 Hilbert space (RKHS) \mathcal{H}_K as $\|\cdot\|_K$ (for details on RKHS see [5]). Given an instance sequence

36 $\mathbf{x} := (x_1, \dots, x_T)$, we let $\mathcal{H}_K^{(x)} := \{h \in \mathcal{H}_K : h(x_t) \in \{-1, 1\}, \forall t \in [T]\}$ denote the functions in
37 \mathcal{H}_K that are binary-valued on the sequence.

38 Here \mathcal{H}_K is real Hilbert space i.e., a set of *real-valued* functions endowed with an inner product and thus a norm
39 (which our bounds are in terms of). In the next sentence we define $\mathcal{H}_K^{(x)} \subset \mathcal{H}_K$ as the subset of functions that are
40 interpolants on the instance sequence \mathbf{x} (observe that if $h \in \mathcal{H}_K$ in the typical case $\text{sign}(h) \notin \mathcal{H}_K$). Since these are
41 linear interpolants not halfspaces this should alleviate your complexity concerns. We provided the reference [5] for
42 the definition of \mathcal{H}_K ; we do not write out the full formal definition in the paper because of its length but we note that
43 RKHSs have been a mainstay of statistical learning theory for at least 30 years.

44 You also misunderstand the implications of infinite *Littlestone* dimension for regret and mistake bounds. Yes for a
45 Gaussian kernel the corresponding spaces have infinite **Ldim**. However, this only rules out a *uniform* bound. One may
46 still have a *non-uniform* bound with respect e.g., to a norm of a given hyp. in the space, see for example Novikoff’s
47 Theorem where the inverse margin corresponds to the norm of the classifier. We provide a non-uniform bound (Thm 3).

48 “*Theorem 50 in Appendix C*” : Please read theorem statement (esp. line 1071 we recall “for any vector \mathbf{u} such that
49 $|\langle \mathbf{u}, \mathbf{x}_t \rangle| = 1$ for $t = 1, \dots, T$.”). The bound is with respect to *interpolants* not *halfspaces*. Note that [33] has a new
50 version in Arxiv, we refer to Lemmas 19 and 20 in version 3. These are minor algebraic and probabilistic results; the
51 only connection to matrix completion is the notation. We will amend the proof to make this clear.