# Multilinear Subspace Regression:
# An Orthogonal Tensor Decomposition Approach

Qibin Zhao[1], Cesar F. Caiafa[2], Danilo P. Mandic[3], Liqing Zhang[4], Tonio Ball[5], Andreas Schulze-Bonhage[5], and Andrzej Cichocki[1]

*[1] Brain Science Institute, RIKEN, Japan*
*[2] IAR, CONICET, Argentina*
*[3] Imperial College, UK*
*[4] Shanghai Jiao Tong University, China*
*[5] Albert-Ludwigs-University, Germany*

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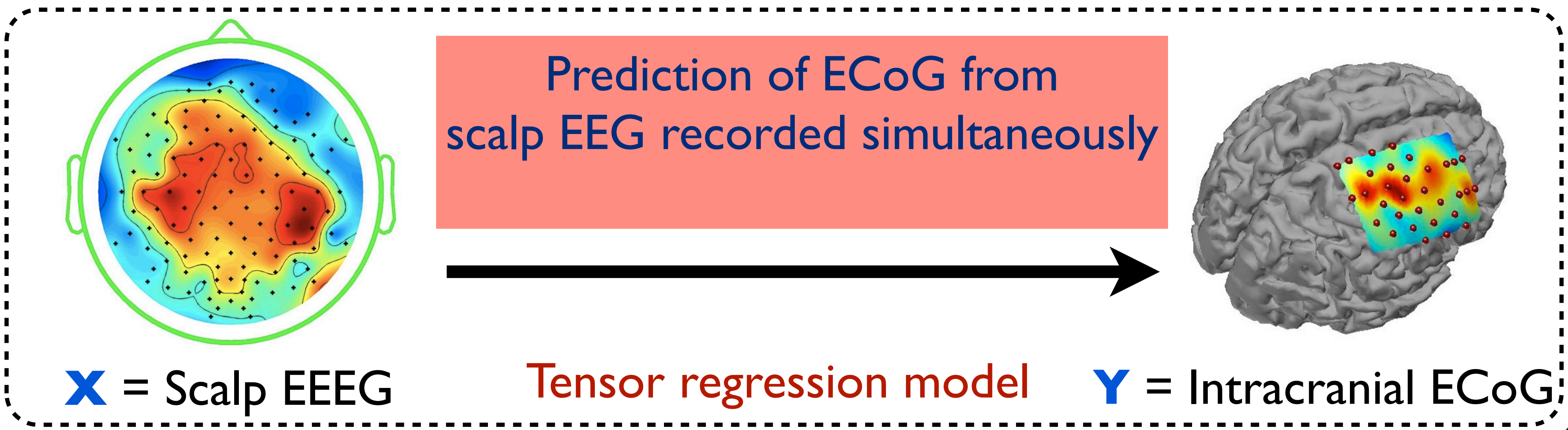
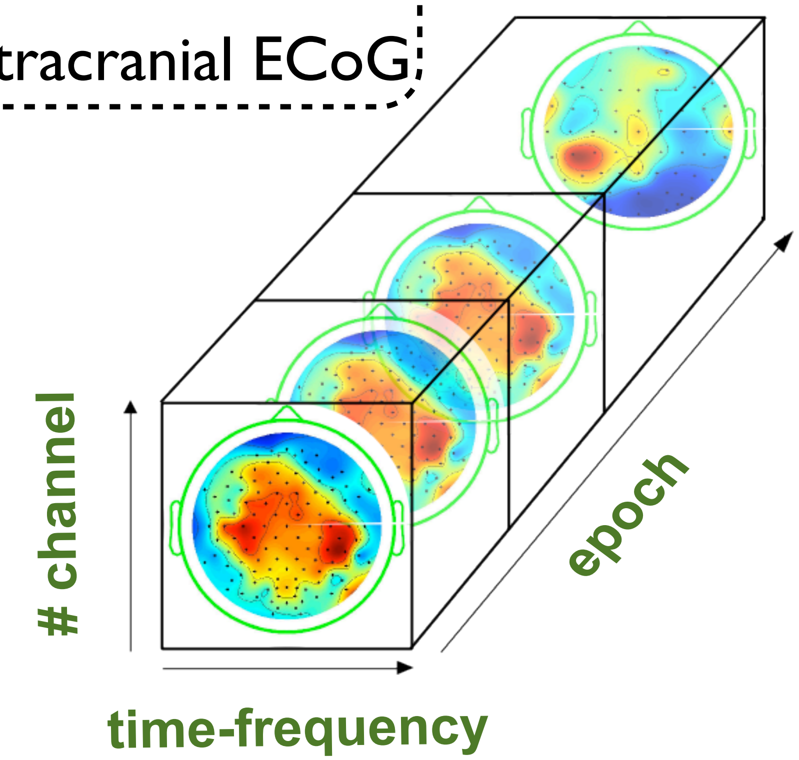
LABSP: http://www.bsp.brain.riken.jp/

# Multilinear regression and applications

- **Tensor representation of multidimensional data**
  - EEG, ECoG (spatial, temporal, frequency, epoch,...)
  - Physical meaning - ease of interpretation
- **From multivariate to multi-way array processes - partial least squares (PLS)**




- **Standard PLS applied on matricization of both X and Y**
  - Small sample size problem
  - Overfitting problem (high dimension of subspace basis)
  - Lack of physical interpretation for loadings

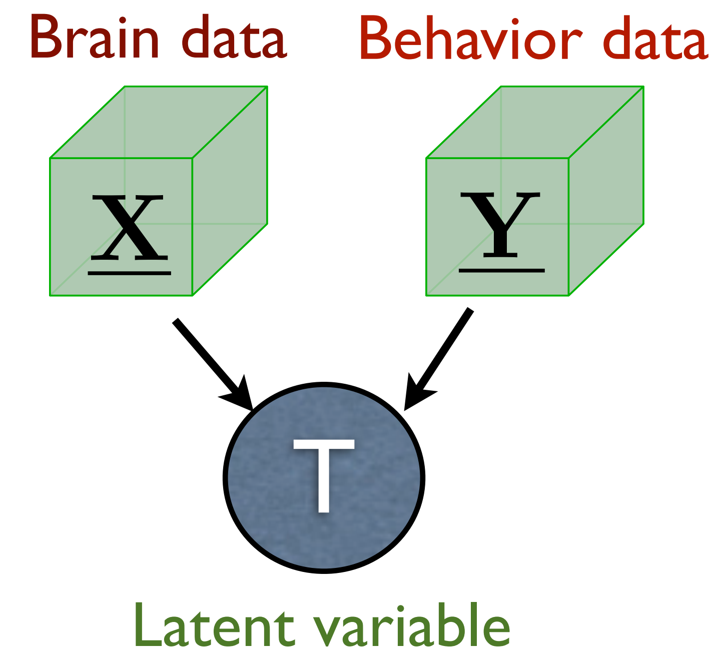


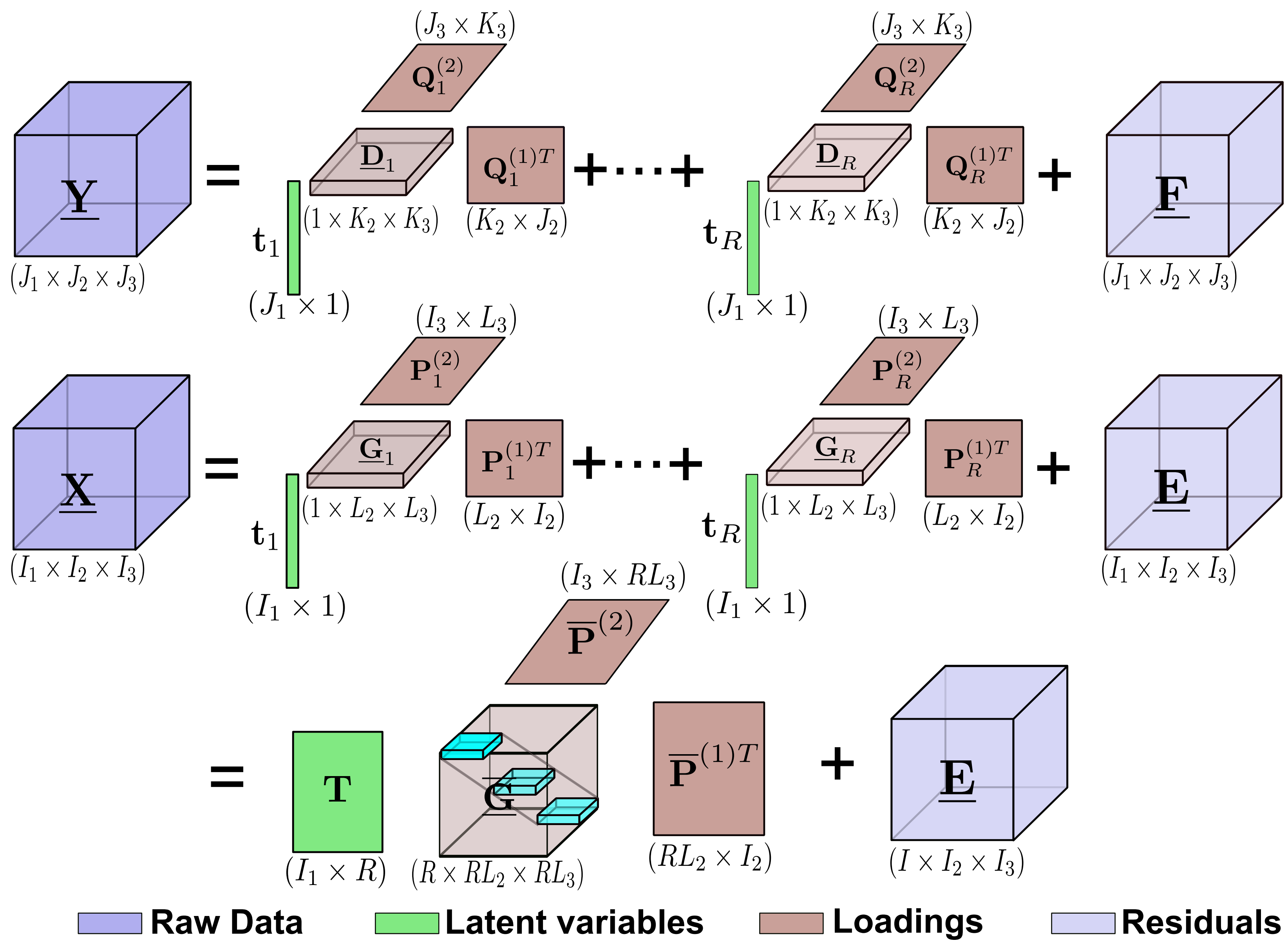
# Proposed approach

**Objective function**

$$\min_{\{\mathbf{P}^{(n)},\mathbf{Q}^{(m)}\}} \left\| \underline{\mathbf{X}} - [\![ \underline{\mathbf{G}}; \mathbf{t}, \mathbf{P}^{(1)}, \ldots, \mathbf{P}^{(N-1)} ]\!] \right\|^2 + \left\| \underline{\mathbf{Y}} - [\![ \underline{\mathbf{D}}; \mathbf{t}, \mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(M-1)} ]\!] \right\|^2$$

$$\text{s. t.} \quad \{\mathbf{P}^{(n)T}\mathbf{P}^{(n)}\} = \mathbf{I}_{L_{n+1}}, \quad \{\mathbf{Q}^{(m)T}\mathbf{Q}^{(m)}\} = \mathbf{I}_{K_{m+1}},$$




**Extension of PLS to higher-order tensor data - HOPLS**

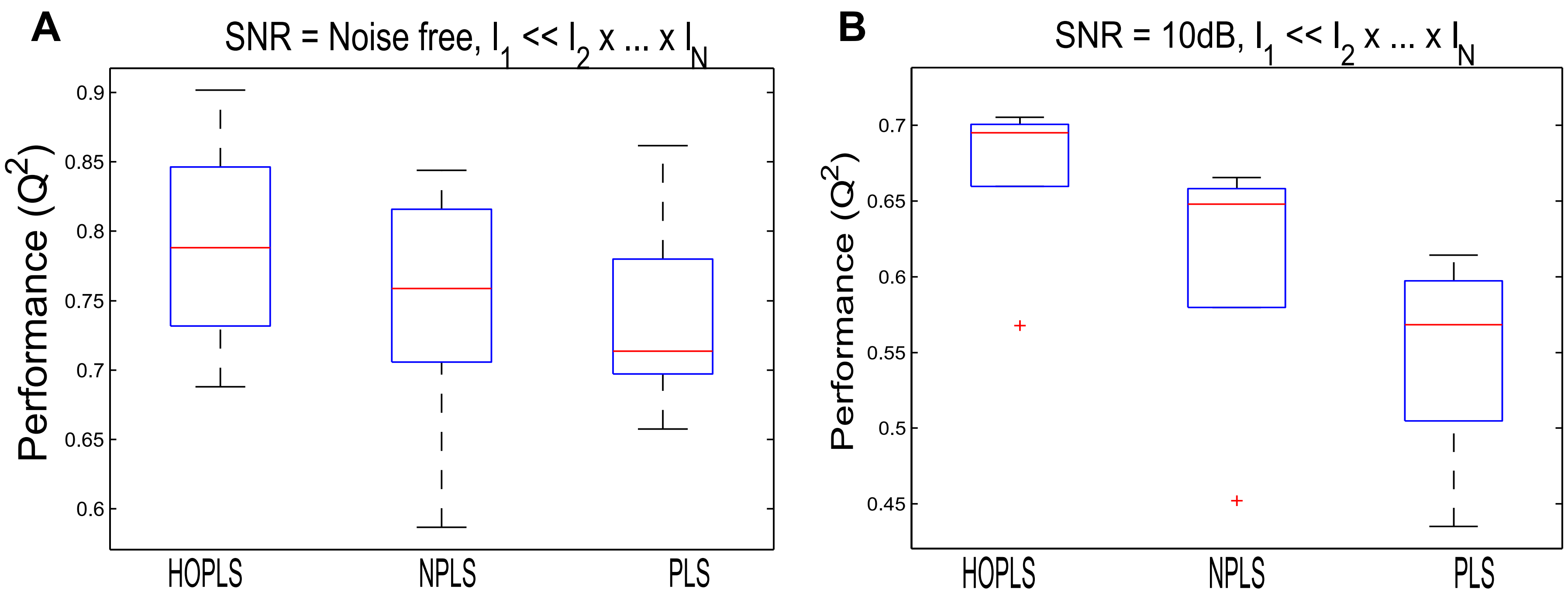
- Goal: to predict a tensor Y from a tensor X
- Approach: to extract the common latent variables

**Properties:**

- Flexible multilinear regression framework
- Projection on tensor subspace basis
- Efficient optimization algorithm using HOOI on the *n*-mode cross-covariance tensor

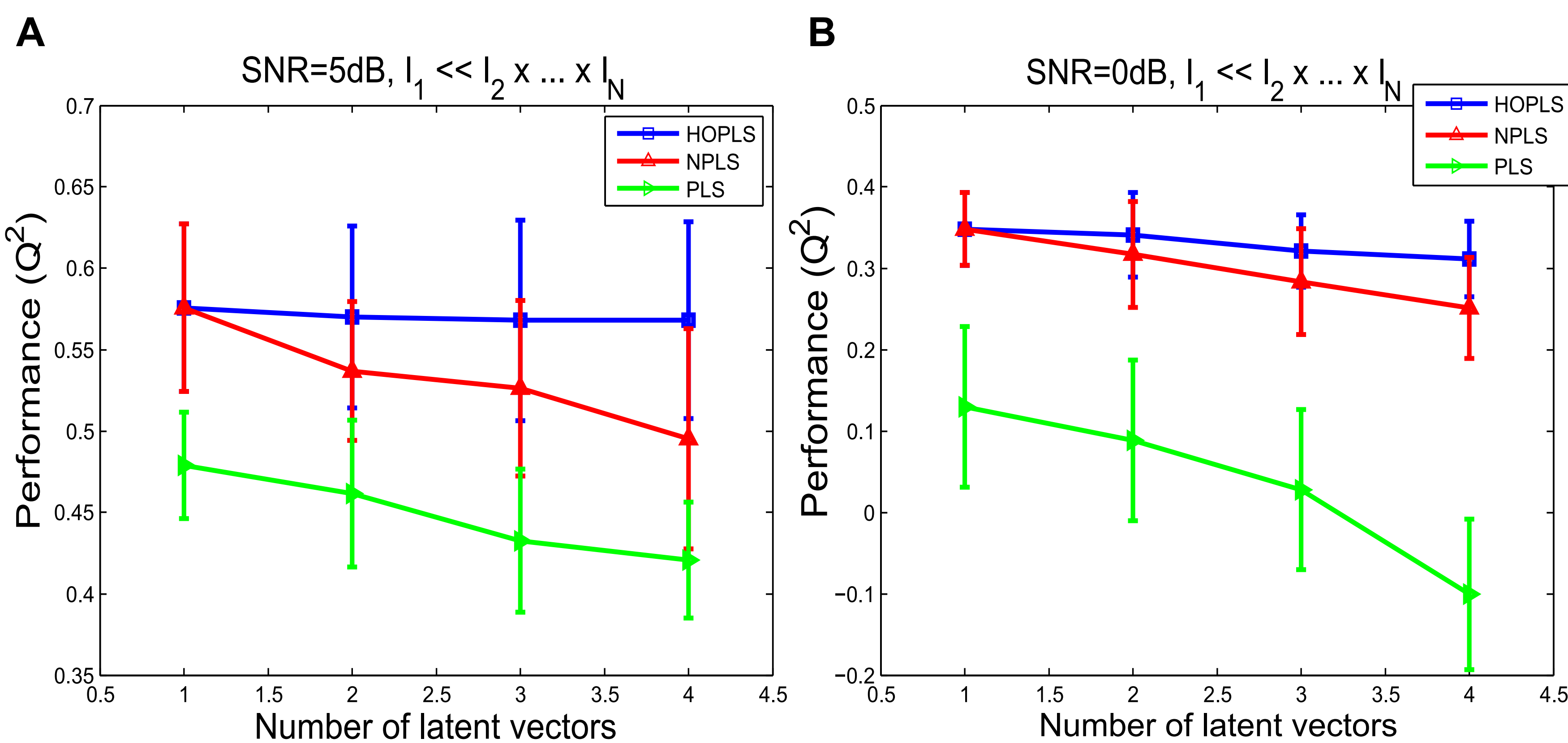
# Key advantages

## Small sample size




HOPLS: better prediction performance and enhanced robustness to noise

## Robustness against overfitting and noise




Stability of the performance of HOPLS, NPLS and PLS for a varying number of latent vectors under different noise conditions

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