Structural equations and divisive normalization for energy-dependent component analysis

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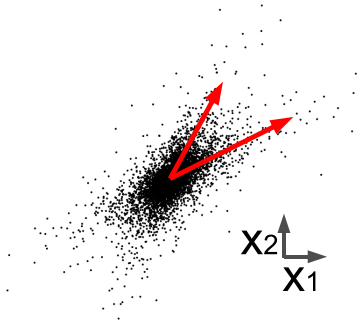
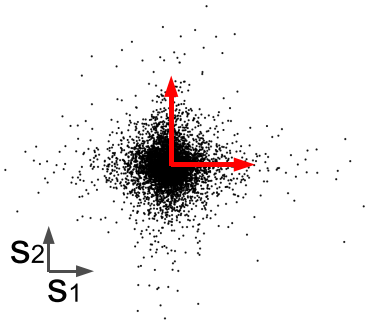
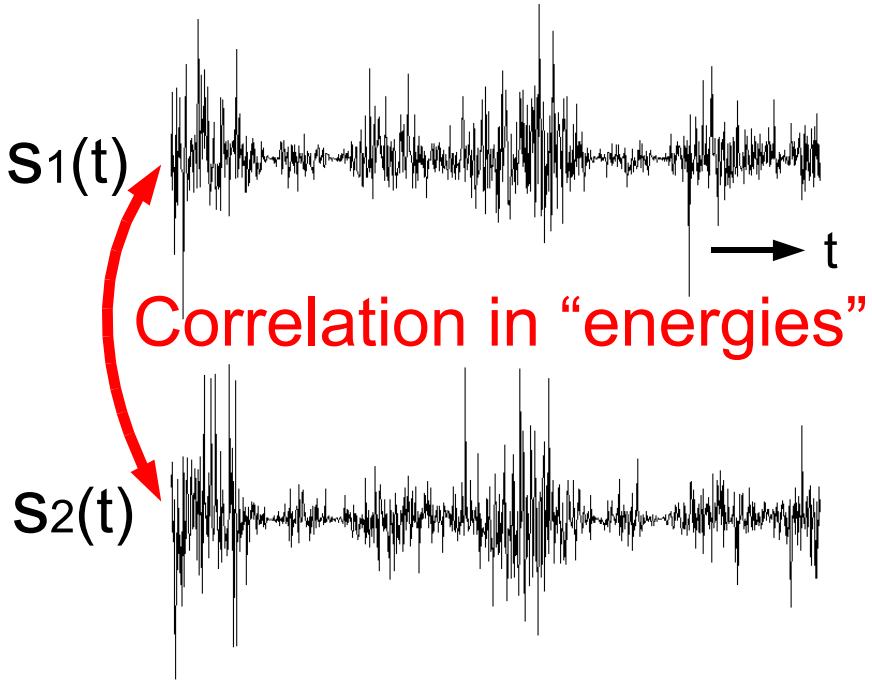
Poster ID: M057

“Energy-dependent” component analysis



$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$$
$$\Longleftrightarrow \mathbf{s}(t) = \mathbf{W}\mathbf{x}(t)$$
$$(\mathbf{W} = \mathbf{A}^{-1})$$



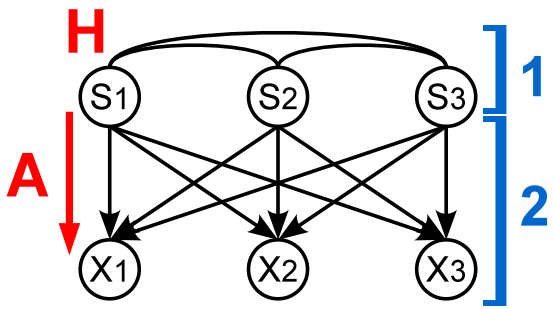


Common characteristics in

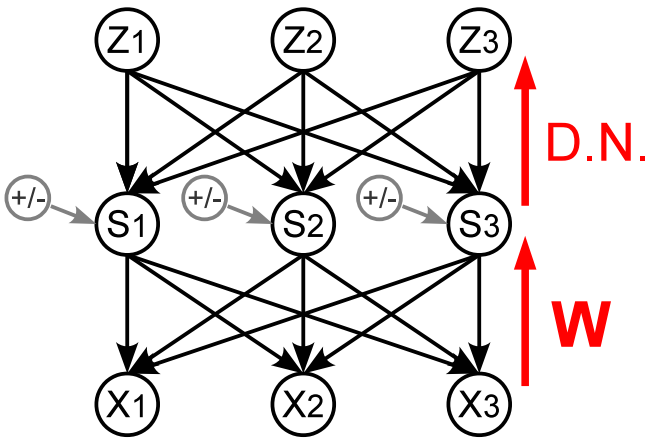
- Natural images/signals
- Brain signals (EEG, MEG, ...)
- Financial time-series etc.

Goal: Jointly estimate energy-interaction and linear mixing

Tractable two-stage generative model



1. Structural equation model (SEM)
   $\ln|s_i(t)| = \sum_j h_{ij} \ln|s_j(t)| + \ln z_i(t)$
   $(i=1,2,\ldots)$
   disturbance
2. Linear mixing $\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$



Inference by *Divisive Normalization*

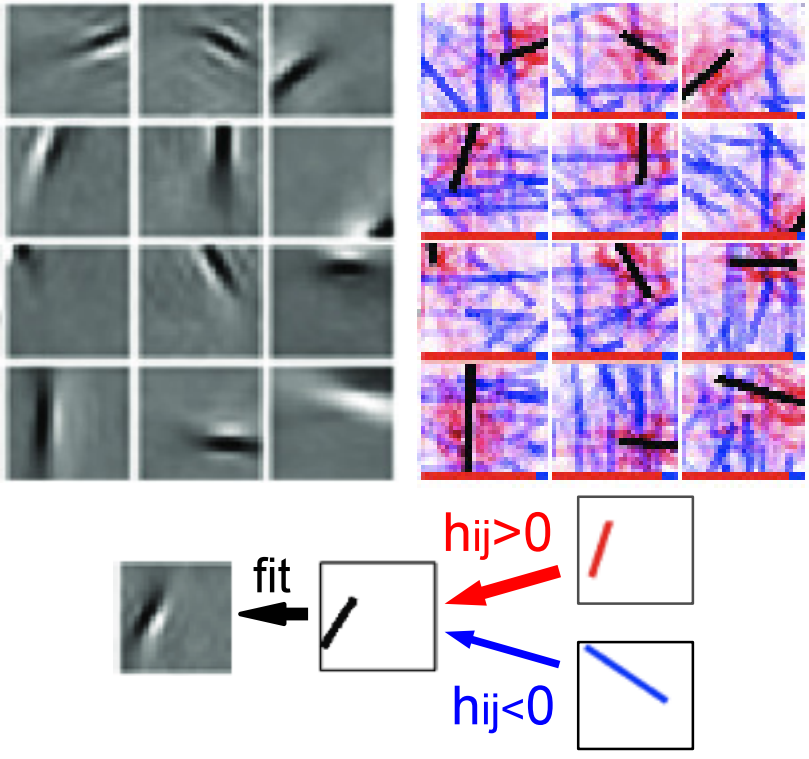
$$z_i(t) = \frac{|s_i(t)|}{\prod_j |s_j(t)|^{h_{ij}}} \qquad \mathbf{s}(t) = \mathbf{W}\mathbf{x}(t)$$

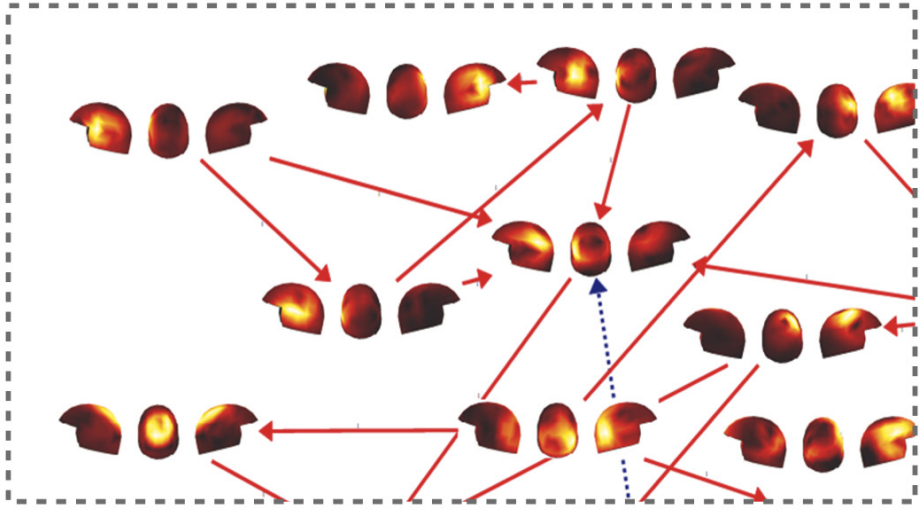
(Conventional DN = Nonlinear SEM)

$$-\ln p(\mathbf{x}|\mathbf{W},\mathbf{V}) = f(\mathbf{V}\psi(\mathbf{W}\mathbf{x})) + g(\mathbf{W}\mathbf{x}) - \ln|\mathbf{W}| - \ln|\mathbf{V}|$$

(f, g: nonlinear functions; $\psi(\cdot) = \ln|\cdot|$; $\mathbf{W} = \mathbf{A}^{-1}$, $\mathbf{V} := \mathbf{I} - \mathbf{H}$)

Results on natural images and MEG signals





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