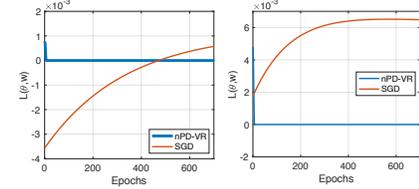


1 The authors would like to thank all the three reviewers for their useful feedback and the area chair for handling this  
 2 paper. To address the reviewers’ comments, upon acceptance of this paper, we will (i) *include numerical experiment*  
 3 *results*, (ii) *provide an explicit bound on  $\alpha$ ,  $\beta$* , and (iii) *improve clarity*. Some common concerns are as follows.

4 **• Empirical Results:** To illustrate the practical performance of nPD-  
 5 VR, we experimented with MountainCar dataset w/  $m = 5000$ . We  
 6 ran Sarsa to obtain a good policy, then we generate a trajectory of the  
 7 state-action pairs. For the nonlinearity, we parametrize  $V$  as a two-layer  
 8 neural network with  $n$  hidden neurons. We set  $\gamma = 0.95$ ,  $\alpha = 10^{-4}$ ,  
 9  $\beta = 10^{-8}$  and constraint as  $\theta \in \Theta = [0, 1]^n$ ,  $w \in [0, 100]^n$ . Trajectory  
 10 of the objective  $\mathcal{L}(\theta^{(k)}, w^{(k)})$  is shown on the right. The objective of  
 11 nPD-VR converges to (close to) zero in 4-5 passes on data, while a single  
 12 timescale SGD on (16) takes a long time (or fail) to converge. Details of this experiment will be found in final version.



(Left)  $n = 50$  neurons (Right)  $n = 100$  neurons.

13 **Reviewer 1:** We thank the reviewer for providing constructive and supportive comments.

14 **Typos:** We apologize for the typos made. **Firstly**, thanks to your suggestions, we have corrected the definition of the  
 15 projection  $\Pi$  and fixed the constant for the cost function. **Secondly**, we clarify that our algorithm only guarantees  
 16 approximate *stationary points* to the *saddle point* MSPBE problem. They will be corrected in the final version.

17 **Assumption 1:** We acknowledge that it is not obvious to check. An intuition is that as  $\Theta$  is bounded and the objective  
 18 is strongly concave in  $w$ , the dual update (w.r.t.  $w$ ) at the  $k$ th iteration pulls the dual variable towards  $w^*(\theta^{(k)})$ , the  
 19 unique maximizer given  $\theta^{(k)}$ , this suggests  $w^{(k)}$  may stay in a bounded set. Details will be provided in the final version.

20 **Complexity:** We will include a comparison to single-timescale primal-dual SGD in terms of computation and memory.  
 21 The per iteration computation complexity for both methods are  $\mathcal{O}(d^2)$  (due to Hessian-gradient mult.), and the memory  
 22 requirement is  $\mathcal{O}(d)$  for SGD and  $\mathcal{O}(md)$  for nPD-VR – we only need to store  $\theta_i^{(k)}, w_i^{(k)}$  as in (20). Convergence  
 23 speed for nPD-VR is  $\mathcal{O}(1/K)$  while the SGD is only anticipated to converge at  $\mathcal{O}(1/\sqrt{K})$  (no known result in the  
 24 literature in this setting). The  $\mathcal{O}(d^2)$  complexity may appear impractical, yet we can apply a diagonal approximation.

25 **Reviewer 2:** We thank the reviewer for providing constructive and supportive comments.

26 **Contributions w.r.t. Related Work:** The suggested references are useful and will be included. We remark that  
 27 [1]-[4] only considered linear function approximation, while this work focuses on the nonlinear setting. Also, the fast  
 28 convergence of primal-dual SAGA for *one-sided non-convex* problem is new even to the optimization community.

29 **Existence of  $\beta$ :** We checked (a0),(a1),(a2),(a3),(a4) carefully and derived this:

$$\alpha \leq \min \left\{ \frac{\mu^2}{8L_w^2 m}, \frac{1/m}{(16L_w^2 + 2)}, \frac{1/m}{12\bar{L}^2 + 96L_w^2/\mu^2} \right\}, \beta \leq \min \left\{ \frac{\mu^2}{48L_w^2} \alpha, \frac{1}{8} \left( 8m\bar{L}^2 \frac{L_w^2}{\mu^2} + \frac{L_\theta}{2} + \frac{\mu^2}{8m} \right)^{-1} \right\}.$$

30  $\bar{L}^2 = 2L_w^2 + L_\theta^2$  and loose constraints are skipped. To get  $\alpha, \beta$ , we first fix  $\alpha$  with the first inequality, then obtain  $\beta$ .

31 **Reviewer 3:** We emphasize that our contributions are substantial, from both TD learning and optimization perspectives:

32 First, we disagree that our paper does not provide a *real analysis for TD algorithm*. Our (10) is actually a TD learning  
 33 problem, and we focus on tackling its **batch/offline version (16)**. While in this setting the randomness in state/action  
 34 becomes decoupled from the learning process, there are **vast applications** related scenarios with offline available data  
 35 and experience replay – as studied in “Batch Reinforcement Learning” by Lange et al., “Least squares policy evaluation  
 36 algorithms with linear function approximation” by Nedić et al., etc. (references will be added). All these works only  
 37 focused on *linear function approximation*, while we study the **nonlinear** case. Nonlinear TD learning is a challenging  
 38 problem due to **non-convexity** in the underlying optimization. It has only been studied by a few authors, e.g., [4,7],  
 39 and there are **no prior finite-time analysis** papers. While we consider the batch/offline setting (16), we developed  
 40 an algorithm with finite-time analysis and is **efficient**. This result is one of the first in the literature and advances the  
 41 analytical understanding for TD learning.

42 We also disagree that our analysis follows from *standard techniques* in optimization. While the use of variance reduction  
 43 (VR) on finite-sum problems is common, applying and analyzing VR on **one-sided non-convex primal dual problem**  
 44 (that arises from nonlinear TD) is **new** and **non-trivial**, even to the optimization community. The only comparable  
 45 results are the recent works [22,24,27] with focuses on two-timescale and batch update methods. Our novelty is also  
 46 evidenced in the analysis in Appendix D where we developed novel analysis techniques to handle the unique challenges.

47 An **online nonlinear TD learning** algorithm, that accounts for Markovian randomness in state/action, is an interesting  
 48 extension. It relates to a stochastic approximation scheme [arXiv:1806.02450] for bilevel programs [arXiv:1802.02246],  
 49 and a finite time analysis is possible. This, however, belongs to a different setting than our focus.