

1 We would like to thank the reviewers for their time, constructive feedback, and critical suggestions that will help us
2 improve the paper. As noted in the reviews, the final submission of the paper needs additional refinements. In particular,
3 based on the reviewers' comments, in the final submission of the paper we will include discussions on the following
4 topics,

- 5 • The conditions (in terms of the number of samples n , and the tuning parameter λ) under which the regularized
6 logistic regression has a unique solution: More specifically, we state that when λ is chosen in a proper range
7 (which depends on the regularizer f), the solution is unique.
- 8 • Interpretations of the variables in the nonlinear system (6): Although some insights have been provided in
9 Section 3.3, we elaborate on that and also briefly illustrate the method to derive this system of equations.
- 10 • More details regarding the setting and the results of our numerical simulations.
- 11 • Conclusions and potential avenues of pursuit (which will be added as a separate section.)

12 Please find our individual responses to each of the reviews in their respective threads.

13 **Reviewer1:** As stated in the comments, the main contribution of this paper is the precise analysis of the asymptotic
14 performance of the regularized logistic regression which generalizes the previous results reported by Sur and Candes [22].
15 We also provide detailed specialization of the result for the ℓ_1 and ℓ_2^2 regularized cases that are used in many practical
16 applications (see [3,14,16, 28] and references therein.)

17 The reviewer is correct that the nonlinear system (6) has been presented somewhat abruptly. In the final submission we
18 will add a discussion to provide some intuitions on this nonlinear system of equations as well as the interpretations
19 of its variables. Our result in its current form assumes the uniqueness of the solution. We can show that in general
20 when the parameter λ is chosen in a proper range, the solution is indeed unique. As an example, it can be observed
21 that as $\lambda \rightarrow \infty$, $\hat{\beta} = \mathbf{0}$ is the unique solution of the regularized logistic regression (5). Also, when the regularizer is
22 strictly convex (e.g. $\|\cdot\|_2^2$) the solution would be unique. We will include a discussion on the uniqueness of the solution
23 of the regularized logistic regression in the final submission. However, the exact characterization of the conditions
24 (on the regularization parameter λ) under which the solution is unique is deferred to future works. We shall note that,
25 for the **unregularized** logistic regression, this problem has been analyzed in [5]. Finally, to address the suggested
26 improvements by the reviewer we will add a section for conclusion and future directions.

27 **Reviewer2:** As pointed out by the reviewer, the results presented in this paper provides a precise analysis for
28 the regularized LR, for any locally Lipschitz performance measure $\Psi(\cdot)$. We thank the reviewer for the suggested
29 improvements. (a) It is worth noting that Section 3.3 provides interpretations on the solution of the nonlinear systems.
30 More specifically, Corollary 1 states that $\bar{\alpha}$ is the bias coefficient, and Corollary 2 establishes that $\bar{\sigma}^2$ denotes the
31 variance. In the final submission, we are going to add a comprehensive discussion providing further intuition on
32 the interpretation of variables. (b) There are multiple approaches for solving a nonlinear system of equations. The
33 fixed-point iterative method explained in Remark 2 is the algorithm that we used in our numerical simulations. A
34 detailed discussion on how to solve a nonlinear system is beyond the scope of this paper, yet we will include some
35 references in the final submission to address the reviewer's point.

36 We appreciate the reviewer's attention to the technical intricacies of our proof. We will provide a discussion on the
37 conditions for the uniqueness of the solution. In short, we can state that, when λ is chosen in a proper range (more
38 generally, when $\lambda > C$, for a constant $C := C(f, n)$ that depends on the regularizer $f(\cdot)$, and the number of samples
39 n), the optimization program (67) has a unique solution. Consequently, we will establish that at the optimal point the
40 parameters σ^* , τ^* , r^* , and ν^* are all positive (> 0). Hence, the first-order optimality condition in (68) would give the
41 optimal value. We also appreciate the typographical comments from the reviewer. We will take care of them in the final
42 version.

43 **Reviewer3:** As stated by the reviewer, this paper provides a novel analysis on the asymptotic performance of the
44 regularized logistic regression, with ℓ_1 and ℓ_2^2 regularization as specific instances. We thank the reviewer for asking
45 intriguing question as well as detecting a typographical error in our writing.

46 1- For a performance metric $\Psi(\cdot)$ and a distribution Π , there exist an optimal value for λ which minimizes the estimation
47 error. In general, the optimal value λ_{opt} can not be expressed in a closed-form. Nonetheless, having the precise
48 characterization on the performance of the RLR, we are able to numerically evaluate this optimal value. In our
49 numerical simulations, for ℓ_2^2 (Figure 1.c) and ℓ_1 regularization (Figure 2.c), we have computed λ_{opt} that minimizes the
50 mean-squared error.

51 2- As mentioned above, we are going to add a discussion providing the conditions under which the solution to the
52 optimization program (and the nonlinear system) is unique. One can show that when λ is chosen in an appropriate
53 range the solution would be unique.

54 3- The assumption that the samples $\{\mathbf{x}_i\}_{i=1}^n$ are independent and normally distributed is critical for our proof, since the
55 CGMT framework relies on that. We must also point out that the previous results (in [22]) have the same assumption
56 that the data matrix \mathbf{X} has iid Gaussian entries. Inspired by the recent universality results, we consider extending the
57 result in this paper to a more general setting. We will add a discussion in that regard in the final submission.

58 We will also correct the typo in line 156 (it should be $\beta \sim \Pi$).