We thank the reviewers for their thorough reviews of our paper. We will fix the typos for the next submission. We want to clarify for all the reviewers that our improved composition result for range-bounded mechanisms applies to all instantiations of the exponential mechanism, and not just those with monotonic quality score. To help clarify, Lemmas 4.1 and 4.3 in our submission states that without any knowledge of the quality score, exponential mechanism is ϵ-DP as well as ϵ-range bounded. If it is known that the quality score is monotonic, then the same exponential mechanism is $\epsilon/2$-DP and $\epsilon/2$-range bounded. In both cases our composition bound will improve on advanced composition and in most cases the optimal DP composition bound. We address the questions and concerns for each reviewer below.

Reviewer 1: As the reviewer pointed out, we were careful to cite the 2015 version rather than the 2018 version of Dwork, Su, Zhang since the one-shot top-k result was removed from the more recent version. However, our result is a strict improvement on the result in the 2015 version. Specifically, by adding Gumbel noise and taking the top-k in one shot, we allow for the output to be a ranked list of elements, as would be the case with a peeling exponential mechanism approach. In DSZ'15, they use Laplace noise to return a set of k elements in one shot and crucially cannot give the ranking. We state this in the introduction of our submission, although we use the incorrect author name "Qiao, et al" despite it being the correct citation. We will fix this in the next submission. We were not aware of a formal citation for the connection between exponential mechanism and Gumbel noise since it might be folklore in the community. However, the connection between peeling-exponential mechanism with a one-shot Gumbel, to our knowledge, has not appeared elsewhere, although we do not view this as one of the primary contributions of this work.

Reviewer 2: We interpret the question regarding sparse-vector as whether Gumbel noise can instead be added to this known algorithm, and then apply our improved composition bound. We think this is an interesting direction of future work, but in our initial attempts, we do not see a natural way for this variant to be range bounded.

Due to space limitations and without a natural comparison with existing top-k DP algorithms (others must return k elements and use a known domain), we did not include experiments. However, we did compare our range bounded composition bound with the existing optimal DP composition bound to show that we can get a significant improvement.

Reviewer 3: We see that the range bounded characterization of our algorithms is a significant contribution of this work. This was crucial in showing that we can improve on, not just advanced composition, but the optimal DP composition. If an algorithm is ϵ-range bounded, one can only conclude it is also ϵ-DP, but one can save almost a factor of two in composition knowing the sequence is also ϵ-range bounded. We were unable to find such a remark in McSherry and Talwar that makes a similar observation to range-boundedness. Perhaps the remark the reviewer is referring to is the distinction of monotonic vs general quality scores, where there is a savings of 2 in the algorithm's DP parameter. On the other hand, range boundedness provides an additional savings in composition, which to our knowledge was not known before. We certainly understand and agree with the reviewer that, from a pure theory perspective, a 50% savings is not substantial. However, if we fix the total epsilon of a privacy system, this savings can translate to a significant increase in the total number of queries, which translates to the usefulness and viability of a product. Given that the exponential mechanism is widely used and fundamental in the DP literature, we felt that this was the appropriate venue for a combination of such a theoretical and practical contribution.

We have also explored the connection of privacy odometers [Rogers et al 2016] to this setting (as mentioned in the Related Works section). However, there is a crucial distinction between their setting and ours. In the odometer setting, the privacy parameter is determined based on the previous outcomes. In the context of random walks, the size of the step at round t is completely determined by the previous steps at round $1, \ldots, t-1$. In our current setting, the privacy loss for a call to the top-k algorithm depends on the size of the outcome at the current round t. Hence, we cannot determine the privacy loss if we condition just on the previous outcomes, since it depends on the current round's randomness. This distinction makes privacy odometers not applicable in our setting. However, as we show, our algorithms can be analyzed as a sequence of range bounded algorithms despite each top-k algorithm returning possibly fewer results. Hence, composition follows from analyzing the full sequence of range bounded mechanisms.

We have not considered the application of this approach to PATE, which is a great direction for future work. One difficulty with directly applying our results is that the privacy loss for the top-1 would be the same whether a $\perp$ or an actual value were returned.

To address the questions raised:

[full paper, page 7] Yes, $v_\perp$ is the threshold, we will fix this in the next submission. The rest of the sentence "any index in the top-$\bar{k}$ ... but not in top-$\bar{k}$ for a neighboring histogram ..." is correct as is with $\bar{k}$.

[full paper, page 29] We used the terms "soundness" and "completeness" since we wanted to directly compare to the utility results in Bhaskar et al. who also use those terms.