Learning to Dispatch for Job Shop Scheduling via Deep Reinforcement Learning

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1 Details of the Training Algorithm

In the paper, we adopt the Proximal Policy Optimization (PPO) algorithm [36] to train our agent. Here we provide details of our algorithm in terms of pseudo code, as shown in Algorithm 1. Similar to the original PPO in [36], we also use N actors, each solves one JSSP instance drawn from a distribution $\mathbb{D}$. The difference to [36] is that, instead of sampling a batch of data, we use all data collected by the N actors to perform update, i.e. line 13, 14, 15, and 19 in the psuedo code.

Algorithm 1: PPO learning to dispatch

Input : actor network π_θ and behaviour actor network $\pi_{\theta_{old}}$, with trainable parameters $\theta_{old} = \theta$; critic network v_ϕ with trainable parameters ϕ; number of training steps U; discounting factor γ; update epoch K; policy loss coefficient c_p; value function loss coefficient c_v; entropy loss coefficient c_e; clipping ratio ϵ.

1 Initialize π_θ, $\pi_{\theta_{old}}$, and v_ϕ ;
2 **for** $u = 1, 2, ..., U$ **do**
3   Draw N JSSP instances from $\mathbb{D}$;
4   **for** $n = 1, 2, ..., N$ **do**
5   **for** $t = 0, 1, 2, ...$ **do**
6   Sample $a_{n,t}$ based on $\pi_{\theta_{old}}(a_{n,t}|s_{n,t})$;
7   Receive reward $r_{n,t}$ and next state $s_{n,t+1}$;
8             $\hat{A}_{n,t} = \sum_0^t \gamma^t r_{n,t} - v_\phi(s_{n,t})$, $r_{n,t}(\theta) = \frac{\pi_\theta(a_{n,t}|s_{n,t})}{\pi_{\theta_{old}}(a_{n,t}|s_{n,t})}$;
9   **if** $s_{n,t}$ *is terminal* **then**
10   break;
11   **end**
12   **end**
13         $L_n^{CLIP}(\theta) = \sum_0^t \min\left(r_{n,t}(\theta)\hat{A}_{n,t}, \text{clip}\left(r_{n,t}(\theta), 1-\epsilon, 1+\epsilon\right)\hat{A}_{n,t}\right)$;
14         $L_n^{VF}(\phi) = \sum_0^t \left(v_\phi\left(s_{n,t}\right) - \hat{A}_{n,t}\right)^2$;
15         $L_n^S(\theta) = \sum_0^t S(\pi_\theta(a_{n,t}|s_{n,t}))$, where $S(\cdot)$ is entropy;
16   Aggregate losses: $L_n(\theta, \phi) = c_p L_n^{CLIP}(\theta) - c_v L_n^{VF}(\phi) + c_e L_n^S(\theta)$;
17   **end**
18   **for** $k = 1, 2, ..., K$ **do**
19   Update θ, ϕ with cumulative loss by Adam optimizer:
           $\theta, \phi = \text{argmax}\left(\sum_{n=1}^N L_n(\theta, \phi)\right)$
20   **end**
21     $\theta_{old} \leftarrow \theta$
22 **end**

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2 Details of the Baselines

In this section, we show how the baseline PDRs compute the priority index for the operations. We begin with introducing the notations used in these rules, summarized as follows:

- Z_{ij}: the priority index of operation O_{ij};
- n_i: the number of operations for job J_i;
- Re_i: the release time of job J_i (here we assume $Re_i = 0$ for all J_i, i.e. all jobs are available in the beginning, but in general the jobs could have different release time);
- p_{ij}: the processing time of operation O_{ij}.

Based on the above notations, the decision principles for each baseline are given below:

- *Shortest Processing Time* (SPT): $\min\ Z_{ij} = p_{ij}$;
- *Most Work Remaining* (MWKR): $\max\ Z_{ij} = \sum_j^{n_i} p_{ij}$;
- *Minimum ratio of Flow Due Date to Most Work Remaining* (FDD/MWKR): $\min\ Z_{ij} = \left(Re_i + \sum_1^j p_{ij}\right) / \sum_j^{n_i} p_{ij}$;
- *Most Operations Remaining* (MOPNR): $\max\ Z_{ij} = n_i - j + 1$.

3 Result on Taillard's Benchmark

Here we present the complete results on Taillard's benchmark. In Table S.1, we report the results of training and testing on 5 groups of instances with sizes up to 30×20. As we can observe from this table, the PDRs trained by our method outperform the baselines on 92% of these instances (46 out of 50). In Table S.2, we report the generalization performance of our polices trained on 20×20 and 30×20 instances. Without training, the two trained PDRs achieve the best performance on all the 30 instances, with the 30×20 policy performing slightly better.

	Instance	SPT	MWKR	FDD/WKR	MOPNR	Ours	UB
15×15	Ta01	1872 (52.1%)	1786 (45.1%)	1841 (49.6%)	1864 (51.4%)	**1443 (17.2%)**	1231*
	Ta02	1709 (37.4%)	1944 (56.3%)	1895 (52.3%)	1680 (35.0%)	**1544 (24.1%)**	1244*
	Ta03	2009 (64.9%)	1947 (59.9%)	1914 (57.1%)	1558 (27.9%)	**1440 (18.2%)**	1218*
	Ta04	1825 (55.3%)	1694 (44.2%)	1653 (40.7%)	1755 (49.4%)	**1637 (39.3%)**	1175*
	Ta05	2044 (67.0%)	1892 (54.6%)	1787 (46.0%)	**1605 (31.1%)**	1619 (32.3%)	1224*
	Ta06	1771 (43.1%)	1976 (59.6%)	1748 (41.2%)	1815 (46.6%)	**1601 (29.3%)**	1238*
	Ta07	2016 (64.3%)	1961 (59.8%)	1660 (35.3%)	1884 (53.5%)	**1568 (27.8%)**	1227*
	Ta08	1654 (35.9%)	1803 (48.2%)	1803 (48.2%)	1839 (51.1%)	**1468 (20.6%)**	1217*
	Ta09	1962 (54.0%)	2215 (73.9%)	1848 (45.1%)	2002 (57.1%)	**1627 (27.7%)**	1274*
	Ta10	2164 (74.4%)	2057 (65.8%)	1937 (56.1%)	1821 (46.7%)	**1527 (23.0%)**	1241*
20×15	Ta11	2212 (63.0%)	2117 (56.0%)	2101 (54.8%)	2030 (49.6%)	**1794 (32.2%)**	1357*
	Ta12	2414 (76.6%)	2213 (61.9%)	2034 (48.8%)	2117 (54.9%)	**1805 (32.0%)**	1367*
	Ta13	2346 (74.7%)	2026 (50.9%)	2141 (59.4%)	1979 (47.4%)	**1932 (43.9%)**	1343*
	Ta14	2109 (56.8%)	2164 (60.9%)	1841 (36.9%)	2036 (51.4%)	**1664 (23.7%)**	1345*
	Ta15	2163 (61.5%)	2180 (62.8%)	2187 (63.3%)	1939 (44.8%)	**1730 (29.2%)**	1339*
	Ta16	2232 (64.1%)	2528 (85.9%)	1926 (41.6%)	1980 (45.6%)	**1710 (25.7%)**	1360*
	Ta17	2185 (49.5%)	2015 (37.8%)	2093 (43.2%)	2211 (51.2%)	**1897 (29.8%)**	1462*
	Ta18	2267 (62.4%)	2275 (63.0%)	2064 (47.9%)	1981 (41.9%)	**1794 (28.5%)**	1396
	Ta19	2238 (68.0%)	2201 (65.2%)	1958 (47.0%)	1899 (42.6%)	**1682 (26.3%)**	1332*
	Ta20	2370 (75.8%)	2188 (62.3%)	2195 (62.8%)	1986 (47.3%)	**1739 (29.0%)**	1348*
20×20	Ta21	2836 (72.7%)	2622 (59.7%)	2455 (49.5%)	2320 (41.3%)	**2252 (37.1%)**	1642*
	Ta22	2672 (67.0%)	2554 (59.6%)	2177 (36.1%)	2415 (50.9%)	**2102 (31.4%)**	1600
	Ta23	2397 (53.9%)	2408 (54.7%)	2514 (61.5%)	2194 (40.9%)	**2085 (33.9%)**	1557
	Ta24	2787 (69.5%)	2553 (55.3%)	2391 (45.4%)	2250 (36.9%)	**2200 (33.8%)**	1644*
	Ta25	2513 (57.6%)	2582 (61.9%)	2267 (42.1%)	**2146 (34.5%)**	2201 (38.0%)	1595
	Ta26	2649 (61.2%)	2506 (52.5%)	2644 (60.9%)	2480 (50.9%)	**2176 (32.4%)**	1643
	Ta27	2707 (61.1%)	2768 (64.8%)	2514 (49.6%)	2298 (36.8%)	**2132 (26.9%)**	1680
	Ta28	2654 (65.6%)	2370 (47.8%)	2330 (45.4%)	2259 (40.9%)	**2146 (33.9%)**	1603*
	Ta29	2681 (65.0%)	2399 (47.6%)	2232 (37.4%)	2367 (45.7%)	**1952 (20.1%)**	1625
	Ta30	2662 (68.1%)	2424 (53.0%)	2348 (48.2%)	2370 (49.6%)	**2035 (28.5%)**	1584

30 × 15	Ta31	2870 (62.7%)	2590 (46.8%)	**2459 (39.4%)**	2576 (46.0%)	2565 (45.4%)	1764*
	Ta32	3097 (73.6%)	2725 (52.7%)	2672 (49.8%)	2830 (58.6%)	**2388 (33.9%)**	1784
	Ta33	2782 (55.3%)	2919 (63.0%)	2766 (54.4%)	2746 (53.3%)	**2324 (29.8%)**	1791
	Ta34	2956 (61.7%)	2826 (54.6%)	2669 (46.0%)	2464 (34.8%)	**2332 (27.6%)**	1828*
	Ta35	2940 (46.5%)	2791 (39.1%)	2525 (25.8%)	2649 (32.0%)	**2505 (24.8%)**	2007*
	Ta36	2933 (61.2%)	2811 (54.5%)	2690 (47.9%)	2666 (46.6%)	**2497 (37.3%)**	1819*
	Ta37	3065 (73.1%)	2719 (53.5%)	2492 (40.7%)	2584 (45.9%)	**2325 (31.3%)**	1771*
	Ta38	2700 (61.4%)	2706 (61.7%)	2425 (44.9%)	2657 (58.8%)	**2302 (37.6%)**	1673*
	Ta39	2698 (50.3%)	2592 (44.4%)	2596 (44.6%)	**2409 (34.2%)**	2410 (34.3%)	1795*
	Ta40	2843 (70.3%)	2601 (55.8%)	2614 (56.6%)	2432 (45.7%)	**2140 (28.2%)**	1669
30 × 20	Ta41	3067 (53.0%)	3145 (56.9%)	2991 (49.2%)	2996 (49.4%)	**2667 (33.0%)**	2005
	Ta42	3640 (87.9%)	3394 (75.2%)	3027 (56.3%)	2995 (54.6%)	**2664 (37.5%)**	1937
	Ta43	2843 (54.0%)	3162 (71.3%)	2926 (58.5%)	2666 (44.4%)	**2431 (31.7%)**	1846
	Ta44	3281 (65.8%)	3388 (71.2%)	3462 (74.9%)	2845 (43.8%)	**2714 (37.1%)**	1979
	Ta45	3238 (61.9%)	3390 (69.5%)	3245 (62.3%)	3134 (56.7%)	**2637 (31.9%)**	2000
	Ta46	3352 (67.1%)	3268 (62.9%)	3008 (50.0%)	2802 (39.7%)	**2776 (38.4%)**	2006
	Ta47	3197 (69.2%)	2986 (58.1%)	2940 (55.6%)	2788 (47.6%)	**2476 (31.1%)**	1889
	Ta48	3445 (77.9%)	3050 (57.5%)	2991 (54.4%)	2822 (45.7%)	**2490 (28.5%)**	1937
	Ta49	3201 (63.2%)	3172 (61.8%)	2865 (46.1%)	2933 (49.6%)	**2556 (30.3%)**	1961
	Ta50	3083 (60.3%)	2978 (54.9%)	2995 (55.7%)	2900 (50.8%)	**2628 (36.7%)**	1923

Table S.1. **Results on Taillard's Benchmark (Part I).** The "UB" column is the best solution from literature, and "*" means the solution is optimal.

Instance		SPT	MWKR	FDD/WKR	MOPNR	Ours (20 × 20)	Ours (30 × 20)	UB
50 × 15	Ta01	4280 (55.1%)	3899 (41.3%)	3851 (39.5%)	3616 (31.0%)	3793 (37.4%)	**3599 (30.4%)**	2760*
	Ta02	4419 (60.3%)	3763 (36.5%)	3734 (35.5%)	3698 (34.2%)	3487 (26.5%)	**3341 (21.2%)**	2756*
	Ta03	3949 (45.3%)	3894 (43.3%)	3394 (24.9%)	3402 (25.2%)	**3106 (14.3%)**	3186 (17.3%)	2717*
	Ta04	3977 (40.1%)	3739 (31.7%)	3603 (26.9%)	3599 (26.8%)	3322 (17.0%)	**3266 (15.0%)**	2839*
	Ta05	4307 (60.8%)	3782 (41.2%)	3664 (36.8%)	3650 (36.2%)	3336 (24.5%)	**3232 (20.6%)**	2679*
	Ta06	4156 (49.4%)	3951 (42.1%)	4016 (44.4%)	3638 (30.8%)	3501 (25.9%)	**3378 (21.5%)**	2781*
	Ta07	4321 (46.8%)	3883 (31.9%)	3720 (26.4%)	3705 (25.9%)	3581 (21.7%)	3471 (17.9%)	2943*
	Ta08	4090 (41.8%)	4476 (55.1%)	3926 (36.1%)	3661 (26.9%)	**3454 (19.7%)**	3732 (29.4%)	2885*
	Ta09	4101 (54.5%)	3751 (41.3%)	3672 (38.3%)	3530 (33.0%)	3441 (29.6%)	**3381 (27.3%)**	2655*
	Ta10	4347 (59.6%)	3940 (44.7%)	3783 (38.9%)	3581 (31.5%)	**3281 (20.5%)**	3352 (23.1%)	2723*
50 × 20	Ta11	4687 (63.4%)	4313 (50.4%)	4142 (44.4%)	3941 (37.4%)	3830 (33.5%)	**3654 (27.4%)**	2868*
	Ta12	4670 (62.8%)	4542 (58.3%)	3897 (35.8%)	4025 (40.3%)	**3617 (26.1%)**	3722 (29.7%)	2869*
	Ta13	4415 (60.3%)	4069 (47.7%)	3852 (39.8%)	3692 (34.0%)	**3397 (23.3%)**	3536 (28.3%)	2755*
	Ta14	4334 (60.4%)	4176 (54.6%)	4001 (48.1%)	3748 (38.7%)	**3275 (21.2%)**	3631 (34.4%)	2702*
	Ta15	4221 (54.9%)	4600 (68.8%)	4062 (49.1%)	3866 (41.9%)	3510 (28.8%)	**3359 (23.3%)**	2725*
	Ta16	4457 (56.7%)	4209 (47.9%)	3940 (38.5%)	3846 (35.2%)	**3388 (19.1%)**	3555 (25.0%)	2845*
	Ta17	4420 (56.5%)	4172 (47.7%)	3974 (40.7%)	3795 (34.3%)	3848 (36.2%)	**3567 (26.3%)**	2825*
	Ta18	4807 (72.7%)	4428 (59.1%)	3857 (38.5%)	4077 (46.4%)	**3514 (26.2%)**	3680 (32.2%)	2784*
	Ta19	4379 (42.6%)	4758 (54.9%)	4349 (41.6%)	4135 (34.6%)	3763 (22.5%)	**3592 (17.0%)**	3071*
	Ta20	4932 (64.7%)	4484 (49.7%)	4147 (38.5%)	4075 (36.1%)	3976 (32.8%)	**3643 (21.6%)**	2995*
100 × 20	Ta21	7841 (43.5%)	6943 (27.1%)	6818 (24.8%)	6601 (20.8%)	6549 (19.9%)	**6452 (18.1%)**	5464*
	Ta22	7655 (47.8%)	7021 (35.5%)	6358 (22.7%)	6191 (19.5%)	5884 (13.6%)	**5695 (9.9%)**	5181*
	Ta23	7510 (34.9%)	7381 (32.6%)	6967 (25.1%)	6758 (21.4%)	**6411 (15.1%)**	6462 (16.1%)	5568*
	Ta24	7451 (39.6%)	6995 (31.0%)	6381 (19.5%)	6090 (14.1%)	5917 (10.8%)	**5885 (10.2%)**	5339*
	Ta25	7360 (36.5%)	7366 (36.6%)	6757 (25.3%)	6611 (22.6%)	6669 (23.7%)	**6355 (17.9%)**	5392*
	Ta26	7909 (48.1%)	7026 (31.5%)	6641 (24.3%)	6554 (22.7%)	6337 (18.6%)	**6135 (14.8%)**	5342*
	Ta27	7456 (37.2%)	7502 (38.0%)	6540 (20.3%)	6589 (21.2%)	6297 (15.8%)	**6056 (11.4%)**	5436*
	Ta28	7400 (37.2%)	6861 (27.2%)	6750 (25.1%)	6313 (17.0%)	6177 (14.5%)	**6101 (13.1%)**	5394*
	Ta29	7743 (44.5%)	7232 (35.0%)	6461 (20.6%)	6665 (24.4%)	6185 (15.4%)	**5943 (10.9%)**	5358*
	Ta30	7321 (41.3%)	6961 (34.3%)	6534 (26.1%)	6151 (18.7%)	6124 (18.2%)	**5892 (13.7%)**	5183*

Table S.2. **Results on Taillard's Benchmark (Part II).** The "UB" column is the best solution from literature, and "*" means the solution is optimal.

4 Result on DMU Benchmark

Similar conclusion can be drawn from results on DMU benchmark. In Table S.3, we report results of training and testing on 4 groups of instances with sizes up to 30 × 20, where our method outperforms baselines over 87.5% (35 out of 40) of these instances. In Table S.4 which focuses on the generalization performance, our policies trained on 20 × 20 and 30 × 20 instances outperform the baselines on 77.5% (31 out of 40) instances.

Size	Instance	SPT	MWKR	FDD/WKR	MOPNR	Ours	UB
20 × 15	Dmu01	4516 (76.2%)	3988 (55.6%)	3535 (37.9%)	3882 (51.5%)	**3323 (29.7%)**	2563
	Dmu02	4593 (69.7%)	4555 (68.3%)	3847 (42.2%)	3884 (43.5%)	**3630 (34.1%)**	2706
	Dmu03	4438 (62.5%)	4117 (50.8%)	4063 (48.8%)	3979 (45.7%)	**3660 (34.0%)**	2731*
	Dmu04	4533 (69.8%)	3995 (49.7%)	4160 (55.9%)	4079 (52.8%)	**3816 (43.0%)**	2669
	Dmu05	4420 (60.8%)	4977 (81.0%)	4238 (54.2%)	4116 (49.7%)	**3897 (41.8%)**	2749*
	Dmu41	5283 (62.7%)	5377 (65.5%)	5187 (59.7%)	5070 (56.1%)	**4316 (32.9%)**	3248
	Dmu42	5354 (57.9%)	6076 (79.2%)	5583 (64.7%)	4976 (46.8%)	**4858 (43.3%)**	3390
	Dmu43	5328 (54.8%)	4938 (43.5%)	5086 (47.8%)	5012 (45.7%)	**4887 (42.0%)**	3441
	Dmu44	5745 (64.7%)	5630 (61.4%)	5550 (59.1%)	5213 (49.5%)	**5151 (47.7%)**	3488
	Dmu45	5305 (62.1%)	5446 (66.4%)	5414 (65.5%)	4921 (50.4%)	**4615 (41.0%)**	3272
20 × 20	Dmu06	6230 (92.0%)	5556 (71.3%)	5258 (62.1%)	4747 (46.3%)	**4358 (34.3%)**	3244
	Dmu07	5619 (84.5%)	4636 (52.2%)	4789 (57.2%)	4367 (43.4%)	**3671 (20.5%)**	3046
	Dmu08	5239 (64.3%)	5078 (59.3%)	4817 (51.1%)	4480 (40.5%)	**4048 (27.0%)**	3188
	Dmu09	4874 (57.6%)	4519 (46.2%)	4675 (51.2%)	4519 (46.2%)	**4482 (45.0%)**	3092
	Dmu10	4808 (61.1%)	4963 (66.3%)	4149 (39.0%)	4133 (38.5%)	**4021 (34.8%)**	2984
	Dmu46	6403 (58.7%)	6168 (52.9%)	**5778 (43.2%)**	6136 (52.1%)	5876 (45.6%)	4035
	Dmu47	6015 (52.7%)	6130 (55.6%)	6058 (53.8%)	5908 (50.0%)	**5771 (46.5%)**	3939
	Dmu48	5345 (42.0%)	5701 (51.5%)	5887 (56.4%)	5384 (43.1%)	**5034 (33.8%)**	3763
	Dmu49	6072 (63.7%)	6089 (64.1%)	5807 (56.5%)	**5469 (47.4%)**	5470 (47.4%)	3710
	Dmu50	6300 (68.9%)	6050 (62.2%)	5764 (54.6%)	5380 (44.3%)	**5314 (42.5%)**	3729
30 × 15	Dmu11	5864 (71.0%)	4961 (44.6%)	4798 (39.9%)	4891 (42.6%)	**4435 (29.3%)**	3430
	Dmu12	5966 (70.7%)	5994 (71.5%)	5595 (60.1%)	4947 (41.5%)	**4864 (39.2%)**	3495
	Dmu13	5744 (56.0%)	6190 (68.2%)	5324 (44.6%)	4979 (35.3%)	**4918 (33.6%)**	3681*
	Dmu14	5469 (61.1%)	5567 (64.0%)	4830 (42.3%)	4839 (42.6%)	**4130 (21.7%)**	3394*
	Dmu15	5518 (65.1%)	5299 (58.5%)	4928 (47.4%)	4653 (39.2%)	**4392 (31.4%)**	3343*
	Dmu51	6538 (56.9%)	6841 (64.2%)	7002 (68.0%)	6691 (60.6%)	**6241 (49.8%)**	4167
	Dmu52	7341 (70.3%)	6942 (61.0%)	6650 (54.3%)	**6591 (52.9%)**	6714 (55.7%)	4311
	Dmu53	7232 (64.6%)	7430 (69.1%)	7170 (63.2%)	6851 (55.9%)	**6724 (53.0%)**	4394
	Dmu54	7178 (64.6%)	**6461 (48.1%)**	6767 (55.1%)	6540 (49.9%)	6522 (49.5%)	4362
	Dmu55	**6212 (45.4%)**	6844 (60.2%)	7101 (66.3%)	6446 (50.9%)	6639 (55.4%)	4271
30 × 20	Dmu16	6241 (66.4%)	5837 (55.6%)	5948 (58.6%)	5743 (53.1%)	**4953 (32.0%)**	3751
	Dmu17	6487 (70.1%)	6610 (73.3%)	6035 (58.2%)	5540 (45.3%)	**5379 (41.0%)**	3814
	Dmu18	6978 (81.5%)	6363 (65.5%)	5863 (52.5%)	5714 (48.6%)	**5100 (32.7%)**	3844*
	Dmu19	5767 (53.1%)	6385 (69.5%)	5424 (43.9%)	5223 (38.6%)	**4889 (29.8%)**	3768
	Dmu20	6910 (86.3%)	6472 (74.4%)	6444 (73.7%)	5530 (49.1%)	**4859 (31.0%)**	3710
	Dmu56	7698 (55.8%)	7930 (60.5%)	8248 (66.9%)	7620 (54.2%)	**7328 (48.3%)**	4941
	Dmu57	7746 (66.4%)	7063 (51.7%)	7694 (65.3%)	7345 (57.8%)	**6704 (44.0%)**	4655
	Dmu58	7269 (54.4%)	7708 (63.7%)	7601 (61.4%)	7216 (53.3%)	**6721 (42.8%)**	4708
	Dmu59	7114 (53.8%)	7335 (58.6%)	7490 (62.0%)	7589 (64.1%)	**7109 (53.7%)**	4624
	Dmu60	8150 (71.4%)	7547 (58.7%)	7526 (58.3%)	7399 (55.6%)	**6632 (39.5%)**	4755

Table S.3. **Results on DMU Benchmark (Part I).** The "UB" column is the best solution from literature, and "*" means the solution is optimal.

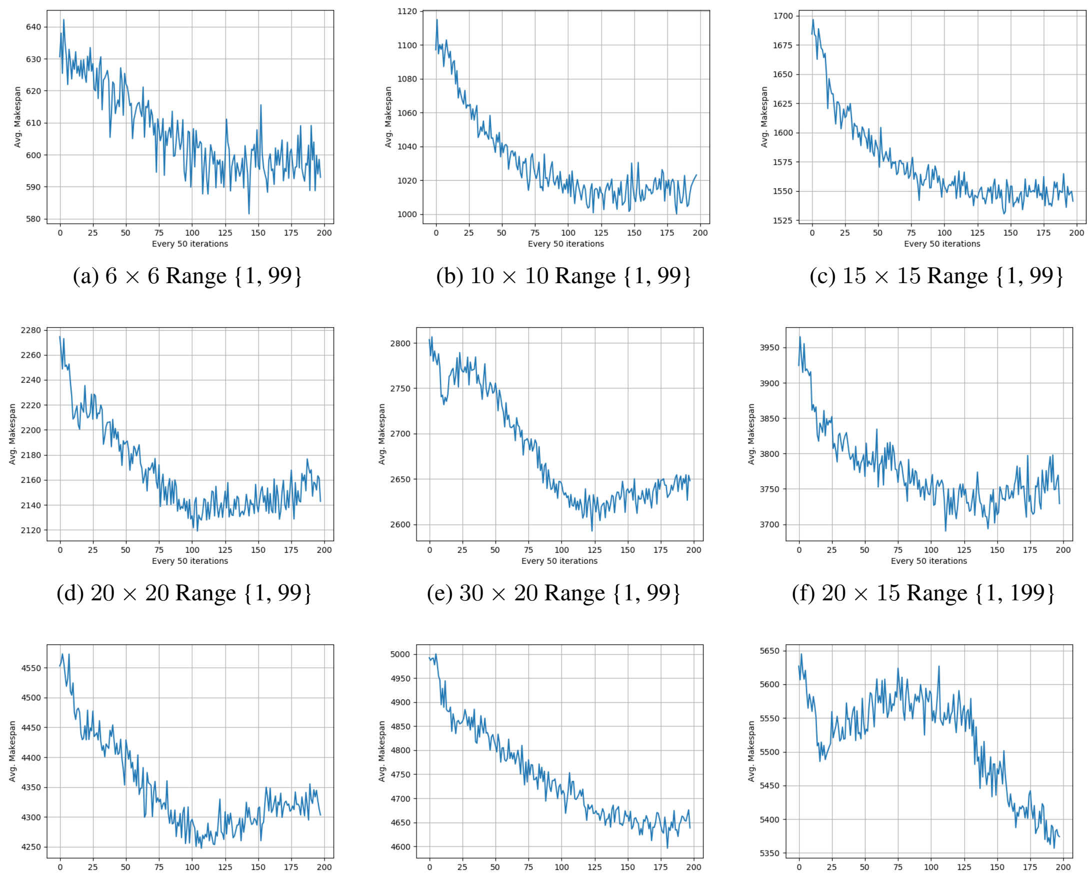
Size	Instance	SPT	MWKR	FDD/WKR	MOPNR	Ours (20 × 20)	Ours (30 × 20)	UB
40 × 15	Dmu21	7400 (68.9%)	6314 (44.2%)	6416 (46.5%)	6048 (38.1%)	5559 (26.9%)	**5317 (21.4%)**	4380*
	Dmu22	7353 (55.6%)	6980 (47.7%)	6645 (40.6%)	6351 (34.4%)	5929 (25.5%)	**5534 (17.1%)**	4725*
	Dmu23	7262 (55.6%)	6472 (38.6%)	6781 (45.3%)	6004 (28.6%)	5681 (21.7%)	**5620 (20.4%)**	4668*
	Dmu24	6799 (46.3%)	7079 (52.3%)	6582 (41.6%)	6155 (32.4%)	**5479 (17.9%)**	5753 (23.8%)	4648*
	Dmu25	6428 (54.4%)	6042 (45.1%)	5756 (38.2%)	5365 (28.8%)	4825 (15.9%)	**4775 (14.7%)**	4164*
	Dmu61	**7817 (51.1%)**	8734 (68.9%)	8757 (69.3%)	8076 (56.1%)	8053 (55.7%)	8203 (58.6%)	5172
	Dmu62	**7759 (47.4%)**	8262 (56.9%)	8082 (53.5%)	8253 (56.8%)	8415 (59.8%)	8091 (53.7%)	5265
	Dmu63	8296 (55.8%)	8364 (57.0%)	8384 (57.4%)	8417 (58.0%)	8330 (56.4%)	**8031 (50.8%)**	5326
	Dmu64	8444 (60.8%)	8406 (60.1%)	8490 (61.7%)	8161 (55.4%)	7916 (50.8%)	**7738 (47.4%)**	5250
	Dmu65	8454 (62.9%)	8189 (57.8%)	8307 (60.1%)	8225 (58.5%)	8093 (55.9%)	**7577 (46.0%)**	5190
40 × 20	Dmu26	7766 (67.1%)	7107 (52.9%)	7240 (55.8%)	6236 (34.2%)	**5908 (27.1%)**	5946 (28.0%)	4647*
	Dmu27	7501 (54.7%)	7313 (50.8%)	6965 (43.7%)	6936 (43.1%)	6542 (34.9%)	**6418 (32.4%)**	4848*
	Dmu28	8621 (83.7%)	8194 (74.6%)	6516 (38.9%)	6714 (43.1%)	6272 (33.7%)	**5986 (27.6%)**	4692*
	Dmu29	8052 (71.6%)	7448 (58.8%)	6971 (48.6%)	6990 (49.0%)	6169 (31.5%)	**6051 (29.0%)**	4691*
	Dmu30	7372 (55.8%)	7890 (66.7%)	6910 (46.0%)	6869 (45.2%)	6022 (27.3%)	**5988 (26.5%)**	4732*
	Dmu66	8971 (56.9%)	8966 (56.8%)	9606 (68.0%)	8726 (52.6%)	8547 (49.5%)	**8475 (48.2%)**	5717
	Dmu67	9096 (56.5%)	9306 (60.1%)	9103 (56.6%)	9372 (61.2%)	**8791 (51.2%)**	8832 (51.9%)	5813
	Dmu68	9265 (60.5%)	9445 (63.6%)	9431 (63.4%)	8722 (51.1%)	9117 (57.9%)	**8693 (50.6%)**	5773
	Dmu69	9215 (61.4%)	9450 (65.5%)	9951 (74.3%)	8697 (52.3%)	9130 (59.9%)	**8634 (51.2%)**	5709
	Dmu70	9522 (61.7%)	9490 (61.1%)	9416 (59.9%)	9445 (60.4%)	**8601 (46.1%)**	8735 (48.3%)	5889

50 × 15	Dmu31	8869 (57.3%)	8147 (44.5%)	7899 (40.1%)	7192 (27.5%)	7191 (27.5%)	**7156 (26.9%)**	5640^*
	Dmu32	7814 (31.8%)	8004 (35.0%)	7316 (23.4%)	7267 (22.6%)	6938 (17.1%)	**6506 (9.8%)**	5927^*
	Dmu33	8114 (41.7%)	7710 (34.6%)	7262 (26.8%)	7069 (23.4%)	6480 (13.1%)	**6192 (8.1%)**	5728^*
	Dmu34	7625 (41.6%)	7709 (43.2%)	7725 (43.5%)	6919 (28.5%)	6661 (23.7%)	**6257 (16.2%)**	5385^*
	Dmu35	8626 (53.1%)	7617 (35.2%)	7099 (26.0%)	7033 (24.8%)	6417 (13.9%)	**6302 (11.8%)**	5635^*
	Dmu71	9594 (53.9%)	9978 (60.1%)	10889 (74.7%)	**9514 (52.6%)**	9950 (59.6%)	9797 (57.2%)	6233
	Dmu72	**9882 (52.4%)**	10135 (56.3%)	11602 (79.0%)	10063 (55.2%)	10401 (60.4%)	9926 (53.1%)	6483
	Dmu73	9953 (61.5%)	9721 (57.7%)	10212 (65.7%)	**9615 (56.0%)**	10080 (63.6%)	9933 (61.2%)	6163
	Dmu74	9866 (58.6%)	10086 (62.2%)	10659 (71.4%)	**9536 (53.3%)**	10445 (67.9%)	9833 (58.1%)	6220
	Dmu75	**9411 (51.9%)**	9953 (60.6%)	10839 (74.9%)	10157 (63.9%)	9937 (60.4%)	9892 (59.6%)	6197
50 × 20	Dmu36	9911 (76.3%)	8090 (43.9%)	8084 (43.8%)	7703 (37.0%)	**7213 (28.3%)**	7470 (32.9%)	5621^*
	Dmu37	8917 (52.4%)	9685 (65.5%)	9433 (61.2%)	7844 (34.1%)	7765 (32.7%)	**7296 (24.7%)**	5851^*
	Dmu38	9384 (64.3%)	8414 (47.3%)	8428 (47.5%)	8398 (47.0%)	7429 (30.0%)	**7410 (29.7%)**	5713^*
	Dmu39	9221 (60.4%)	9266 (61.2%)	8177 (42.3%)	7969 (38.7%)	7168 (24.7%)	**6827 (18.8%)**	5747^*
	Dmu40	9406 (68.7%)	8261 (48.1%)	7773 (39.4%)	8173 (46.5%)	7757 (39.1%)	**7325 (31.3%)**	5577^*
	Dmu76	11677 (71.4%)	10571 (55.2%)	11576 (69.9%)	11019 (61.7%)	10322 (51.5%)	**9698 (42.3%)**	6813
	Dmu77	**10401 (52.5%)**	11148 (63.4%)	11910 (74.6%)	10577 (55.0%)	10729 (57.3%)	10693 (56.7%)	6822
	Dmu78	10585 (56.4%)	10540 (55.7%)	11464 (69.3%)	10989 (62.3%)	10742 (58.7%)	**9986 (47.5%)**	6770
	Dmu79	11115 (59.5%)	11201 (60.7%)	11035 (58.3%)	**10729 (53.9%)**	10993 (57.7%)	10936 (56.9%)	6970
	Dmu80	10711 (60.2%)	10894 (62.9%)	11116 (66.3%)	10679 (59.7%)	10041 (50.2%)	**9875 (47.7%)**	6686

Table S.4. **Results on DMU Benchmark (Part II).** The "UB" column is the best solution from literature, and "*" means the solution is optimal.

5 Training Curve

We show training curves for all problems in Figure.1. The problem sizes are $\{6 \times 6, 10 \times 10, 15 \times 15, 20 \times 15, 20 \times 20, 30 \times 15, 30 \times 20\}$ respectively. In each curve, after learning on every 200 totally new instances, the averaged performance (makespan) over these 200 instances is plotted. The training time for each problem is: 0.95h (for 1a), 2.6h (for 1b), 6.2h (for 1c), 11.6h (for 1d), 20.3h (for 1e), 8.7h (1f), 11.6h (1g), 13.5h (1h), and 20.3h (1i) respectively.



(a) 6×6 Range $\{1, 99\}$

(b) 10×10 Range $\{1, 99\}$

(c) 15×15 Range $\{1, 99\}$

(d) 20×20 Range $\{1, 99\}$

(e) 30×20 Range $\{1, 99\}$

(f) 20×15 Range $\{1, 199\}$

(g) 20×20 Range $\{1, 199\}$

(h) 30×15 Range $\{1, 199\}$

(i) 30×20 Range $\{1, 199\}$

Figure 1: **Training curves for all problems.** The scale of processing time of each problem is given in braces, e.g. $\{1, 99\}$ indicating the scale of processing time is a integer uniformly distributed in range from 1 to 99. Sizes 20×20 and 30×20 have 2 different scales $\{1, 99\}$ and $\{1, 199\}$ respectively.