# Supplemental Material: Meta-learning from Tasks with Heterogeneous Attribute Spaces

#### Tomoharu Iwata

NTT Communication Science Laboratories tomoharu.iwata.gy@hco.ntt.co.jp

## Atsutoshi Kumagai

NTT Software Innovation Center atsutoshi.kumagai.ht@hco.ntt.co.jp

## 1 Experiments

### 1.1 OpenML data

**Data** Table 1 shows the statistics of the OpenML datasets.

**Comparing methods** DS predicted responses by  $\hat{y} = g_{\rm DS}(\frac{1}{I}\sum_{i=1}^{I}f_{\rm DS}(x_i))$ , where  $g_{\rm DS}$  and  $f_{\rm DS}$  were three-layered feed-forward neural networks with 32 hidden units, and the number of output units of  $f_{\rm DS}$  was 32. DS, which can handle different numbers of attributes, was trained to minimize the loss on the training datasets.

With NP, we used deep sets for handling tasks with heterogeneous attribute spaces. In particular, the attributes were encoded in a 32-dimensional vector by a deep set, and the responses were encoded in a 32-dimensional vector by a neural network. Then the two encoded vectors were concatenated and input to a neural network. By averaging the outputs over the instances in the support set, we obtained a task representation. The task representation and each attribute value was concatenated, and the response was predicted by a deep set by taking the set of concatenated vectors as input. In the NP, we used three-layered feed-forward neural networks with 32 hidden units for all the neural networks, and average pooling for all the deep sets.

DS+FT (NP+FT) was the DS (NP) fine-tuned with each target dataset. With DS+MAML (NP+MAML), DS (NP) was trained to minimize the loss on the query sets when fine-tuned with the support sets with model-agnostic meta-learning [1], where support and query sets were sampled in the same way with the proposed method in Algorithm 1. The number of fine-tuning epochs was five.

Ridge, Lasso, BR, KR, GP, NN, and Mean were trained with the target datasets since they cannot handle tasks with heterogeneous attribute spaces. We used the implementation of scikit-learn [2] for them, where the default parameter settings of the scikit-learn were used since the number of support instances was three, which is too small to conduct a cross-validation.

For classification experiments, we used the cross-entropy loss with DS, DS+FT, DS+MAML, NP, NP+FT, NP+MAML, and the proposed method. We used the implementation of scikit-learn [2] with KNN, DT, RF, NN, AB, NB, and MF, where the default parameter settings of the scikit-learn were used.

**Results** Table 2 shows the mean squared error for each target task.

## References

[1] E. Grefenstette, B. Amos, D. Yarats, P. M. Htut, A. Molchanov, F. Meier, D. Kiela, K. Cho, and S. Chintala. Generalized inner loop meta-learning. *arXiv preprint arXiv:1910.01727*, 2019.

Table 1: Statistics of the OpenML datasets used in our experiments.

Index	Name	#instances	#attributes
0	vineyard	52	3
1	bolts	40	7
2	sleep	62	8
3	autoPrice	159	16
4	detroit	13	14
5	longley	16	7
6	diabetes_numeric	43	3
7	baskball	96	5
8	pyrim	74	28
9	gascons	27	5
10	pwLinear	200	11
11	machine_cpu	209	7
12	qsbr_y2	25	10
13	qsfsr1	20	10
14	qsfsr2	19	10
15	qsbralks	13	22
16	qsartox	16	24
17	qsabr2	15	10
18	hip	54	8
19	analcatdata uktrainacc	31	16
20	mu284	284	10
21	pollution	60	16
22	transplant	131	3
23	bodyfat	252	15
24	fri_c0_250_5	250	6
25	fri_c3_100_10	100	11
26	fri_c2_100_5	100	6
27	fri c3 250 10	250	11
28	fri_c2_250_25	250	26
29	fri_c4_100_25	100	26
30	sleuth ex1714	47	8
31	rabe_265	51	7
32	rabe_266	120	3
33	chscase_demand	27	11
34	visualizing_slope	44	4
35	visualizing_environmental	111	4
36	chscase funds	185	2
37	rabe_166	40	2
38	sleuth_ex1605	62	6
39	chscase_vine1	52	10
40	rabe_131	50	6
41	diggle_table_a1	48	5
42	chatfield_4	235	13
43	hutsof99 child witness	42	16
43	rabe_176	70	4
45	visualizing_hamster	73	6
43 46	rabe_148	66	6
47	visualizing_ethanol	88	3
48	chscase_geyser1	222	3
49	humans_numeric	75	15
50	USCrime	73 47	13
51	ICU	200	20
52		150	5
53	EgyptianSkulls	270	14
53 54	heart	270 86	14
54 55	treepipit	86 154	
	edm		18
56 57	slump	103	10
57 58	branin	225 132	3 8
	echocardiogram-uci	132	8

[2] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, et al. Scikit-learn: Machine learning in Python. *Journal of Machine Learning Research*, 12:2825–2830, 2011.

Table 2: Averaged mean squared errors and standard errors on each target task with the OpenML datasets: Values in bold typeface are not statistically different at 5% level from the best performing method in each row according to a paired t-test. The bottom row is the number of tasks where the method achieved the performance that was not statistically different from the best performing method.

Index	Ours	DS	DS+	DS+	NP	NP+	NP+	Ridge	Lasso	BR	KR	GP	NN	Mean
			FT	MAML		FT	MAML							
0	0.827	0.494	0.502	0.516	0.472	0.548	0.463	0.727	1.188	0.623	0.574	1.175	0.917	1.270
1	0.786	0.884	0.903	0.895	0.886	0.876	0.868	1.362	1.490	1.386	0.512	0.903	0.869	1.501
2	0.705	1.737	1.614	1.177	1.051	0.776	0.920	1.188	1.245	1.791	0.851	0.997	1.320	1.217
3	0.710	0.763	0.731	0.556	0.485	0.517	0.552	0.821	1.204	0.769	0.414	0.962	0.855	1.272
4	0.699	0.787	0.794	0.716	0.666	0.695	0.647	1.095	1.407	1.414	0.453	0.931	0.683	1.521
5	0.758	0.362	0.306	0.182	0.140	0.198	0.132	0.267	0.925	0.258	0.084	0.530	0.313	1.138
6	0.689	0.667	0.675	0.697	0.662	0.771	0.675	1.410	1.488	2.445	0.906	1.102	1.305	1.505
7	0.817	0.912	0.929	0.962	0.980	1.117	0.955	1.346	1.557	1.551	1.062	1.024	1.380	1.560
8	0.930	0.919	0.917	0.891	0.854	0.894	0.901	1.180	1.232	1.182	0.952	1.016	1.250	1.232
9	0.707	0.564	0.471	0.407	0.405	0.414	0.459	0.477	1.209	4.763	0.355	0.311	0.349	1.240
10	0.834	0.949	0.979	0.980	0.964	1.055	0.986	1.355	1.477	1.353	0.842	0.992	1.191	1.477
11	0.822	0.440	0.378	0.317	0.279	0.302	0.272	0.577	0.971	0.520	0.375	0.897	0.553	1.551
12	0.803	1.175	1.178	1.105	1.031	1.101	1.044	1.304	1.373	1.607	1.169	0.966	1.311	1.434
13	0.810	1.162	1.173	1.142	1.199	1.234	1.173	1.316	1.378	1.333	1.148	1.057	1.154	1.367
14	0.825	1.232	1.233	1.211	1.268	1.301	1.276	1.060	1.355	1.029	0.724	1.056	0.967	1.355
15	0.728	1.100	1.115	1.154	1.095	1.136	1.096	1.838	1.120	2.129	1.210	1.073	1.798	1.142
16	0.931	0.930	0.950	0.892	0.861	0.948	0.868	1.115	1.457	1.101	0.725	1.067	1.207	1.482
17	0.567	1.106	1.142	1.177	1.145	1.323	1.152	1.007	1.257	1.023	0.644	1.018	0.973	1.300
18	0.831	0.904	0.913	0.907	0.940	1.038	0.938	0.963	1.095	0.858	0.727	0.869	1.093	1.135
19	0.723	0.548	0.559	0.585	0.547	0.629	0.580	0.968	1.014	0.959	0.606	0.903	1.169	1.621
20	0.780	1.210	1.170	1.221	1.194	1.327	1.224	0.978	1.190	1.130	0.731	0.833	1.066	1.242
21	0.688	0.969	0.984	0.977	0.965	1.061	0.976	1.116	1.340	1.192	0.812	0.978	0.934	1.355
22	0.785	0.457	0.371	0.264	0.223	0.362	0.212	0.516	1.155	0.712	0.306	0.644	0.564	1.234
23	0.744	0.746	0.758	0.762	0.738	0.847	0.801	1.080	1.092	1.135	0.463	0.974	0.695	1.209
24	0.795	0.709	0.678	0.660	0.686	0.620	0.630	1.005	1.194	1.029	0.790	0.908	0.889	1.224
25	0.834	0.993	1.000	1.006	1.012	1.067	1.018	1.412	1.187	1.576	1.319	1.006	1.425	1.185
26	0.752	1.278	1.226	1.045	1.151	1.182	1.057	1.451	1.078	1.732	1.293	0.947	1.411	1.051
27	0.676	0.986	0.980	0.990	1.019	1.148	1.020	1.434	1.097	1.560	1.264	0.994	1.324	1.098
28	0.797	1.061	1.068	1.044	1.045	1.144	1.030	1.184	1.199	1.190	1.081	1.001	1.323	1.183
29	<b>0.660</b> 0.706	1.014	1.025	1.020	1.007	1.030	1.011	1.323	1.337 1.275	1.332	0.943 <b>0.523</b>	0.997 0.881	1.169	1.292
30		1.000	1.015	1.027	1.038	1.112	1.028	0.989		1.251			0.664	1.284
31 32	0.834	0.929 0.810	0.930 0.821	0.954 0.874	0.956 0.833	1.069 0.996	0.955	1.921 0.481	1.488 1.283	2.271 <b>0.117</b>	1.509 0.185	1.125 0.482	1.747 0.330	1.351
33	1.014 0.718	0.310					0.853	0.200	1.384	0.086	0.183	0.482	0.330	1.432 1.506
34	0.718	1.619	0.215 1.560	0.090 1.371	0.050 1.398	0.048 1.211	0.067 1.367	0.200	1.119	0.080	0.021	0.575	0.218	1.142
35	0.781	1.740	1.688	1.401	1.316	1.194		1.086	1.272	1.864	1.023	1.097	1.331	1.321
36	0.781	1.740	1.184	1.121	1.180	1.194	1.335 1.180	2.062	1.462	4.445	1.023	1.846	2.200	1.462
37	0.969	0.406	0.368	0.252	0.314	0.360	0.280	0.773	1.209	1.273	0.305	8.921	0.521	1.283
38	0.747	0.708	0.735	0.725	0.702	0.777	0.718	1.109	1.249	1.173	0.814	1.012	1.075	1.252
39	0.834	0.758	0.733	0.723	0.888	0.969	0.881	1.102	1.170	1.263	0.843	0.989	1.137	1.194
40	0.853	0.752	0.757	0.839	0.862	1.048	0.869	0.724	1.153	0.832	0.423	0.732	0.692	1.368
41	0.721	0.839	0.857	0.875	0.855	0.958	0.862	1.577	1.474	2.118	1.221	1.009	1.420	1.488
42	0.950	0.390	0.368	0.297	0.299	0.333	0.303	0.511	1.125	0.483	0.328	0.851	0.620	1.156
43	0.782	0.894	0.901	0.946	0.912	1.050	0.934	1.539	1.117	1.643	1.026	0.993	1.284	1.232
44	0.762	0.823	0.826	0.940	0.912	1.289	0.964	2.305	1.318	2.123	1.341	1.559	2.151	1.232
45	0.845	1.113	1.149	1.102	1.033	1.082	1.023	2.143	1.675	3.024	1.422	1.161	2.003	1.675
46	0.746	0.540	0.528	0.416	0.491	0.694	0.588	1.257	1.347	1.431	0.609	0.947	1.024	1.446
47	0.774	1.195	1.211	1.078	1.130	1.270	1.101	2.037	1.467	2.648	1.519	1.950	2.695	1.461
48	0.828	0.704	0.702	0.726	0.700	0.840	0.705	0.776	1.177	1.227	0.380	0.760	0.753	1.238
49	0.735	1.115	1.135	1.178	1.183	1.241	1.184	1.694	1.191	2.103	1.722	1.098	1.559	1.183
50	0.819	1.141	1.135	1.060	1.081	1.123	1.054	0.848	1.154	0.883	0.509	0.927	0.649	1.412
51	0.861	0.976	0.999	0.988	0.961	1.031	0.968	1.071	1.104	1.071	1.006	1.008	1.257	1.118
52	0.828	1.111	1.113	1.068	1.050	1.109	1.051	1.756	1.389	2.434	1.191	1.030	1.473	1.378
53	0.660	0.816	0.810	0.741	0.754	0.772	0.758	1.528	1.664	1.552	0.748	0.990	1.061	1.664
54	0.934	1.048	1.056	1.050	1.044	1.065	1.033	2.262	2.039	2.832	1.450	0.985	1.563	2.039
55	0.857	0.984	0.982	1.011	1.000	1.039	1.004	1.755	1.116	2.047	1.105	1.023	1.311	1.141
56	0.767	1.061	1.061	1.041	1.039	1.066	1.046	1.441	1.398	1.486	1.010	0.973	1.207	1.397
57	0.775	1.011	0.959	1.050	1.075	1.119	1.087	1.353	1.253	5.520	1.173	0.796	1.161	1.253
58	0.924	0.795	0.813	0.814	0.811	0.902	0.804	1.331	1.384	1.393	1.098	0.980	1.271	1.495
#best	29	2	0	3	4	1	3	0	0	1	15	1	0	0