We thank the reviewers for their thoughtful and helpful feedback. We carefully went through the paper again in the past few days and fixed all the typos and grammatical issues mentioned in the reviews. Below we will address the comments regarding our results and techniques.

Reviewer 1   Thank you again for your insightful and positive comments.

The problem should be a little better motivated. Agree. We motivated the problem in multiple ways – as a novel problem in functional estimation (line 14 to 21), as a generalization of the well-known Fisher's unseen species problem (line 22 to 57), and as an interesting task with numerous practical applications (line 65 to 74). Nevertheless, we want to motivate the problem even better by adding examples of genetic research and scores of basketball players to the introduction. We will also add the respective real-data experiments to the appendices.

Technical novelty Yes, our estimator's construction makes use of some prior results in [15], [16], and [22]. The combination might seem relatively simple in retrospect, but requires someone first to have the insight and realize the potential of the methods. In particular, one needs to properly manipulate each estimator component and provide tight analyses for both the upper and lower bounds. Besides, we made an effort to simplify the estimator (Appendix G) and multiple proofs (e.g., Appendices F and H) for technical beauty.

Reviewer 2   Thank you for the thoughtful and encouraging feedback, and for pointing out the importance of understanding how a large μ value will affect the practical performance of the estimator. The theoretical analysis already showed that the variance will become larger as μ grows. We will add additional experiments for the large-μ regime and comment on the results accordingly.

Reviewer 3   Thank you for the thorough and helpful comments. We really appreciate your effort in helping us correct the typos and improve the clarity of the theorems.

Bounds on universal constants Currently, the lower-bound constant c_0' is 0.4, and the upper-bound constant c_0 is 3.0. These might not be the best constants we can get because they were chosen to simplify the proofs. We will make all the four constants (c, c_0, c', and c_0') explicit in the draft and also work on optimizing their values.

Reviewer 4   Thank you for your constructive and valuable insights.

Replacing the word "reproducibility" Yes, we agree with your opinion on the choice of words. Thanks for also providing the references on the word "reproducibility" and its usage in the scientific communities. We have modified our paper accordingly and replaced "reproducibility" by "multiplicity."

Claim on "resolv(ing) this problem" The claim simply refers to the fact that the lower bound matches the upper bound, up to constants in the exponents. The theorems require $\mu = \mathcal{O}(\log n)$, which translates to something like $a \geq 1$ for $a \approx \log n/\mu$. Otherwise, the problem becomes relatively simple as we extrapolate no more than what has been observed. The condition of $a \geq 1$ was also required in our primary references [16, 22] and termed as "the interesting case" in the arXiv paper you mentioned. We appreciate the sharp observation and have updated the draft accordingly.

Lower bound proof and support size estimation Here is an over-simplified yet intuitive explanation. The paper [32] on support size estimation constructed nontrivial lower bounds for $m/\log m \ll k \ll m\log m$, where m is the sample size and $1/k$ is a lower bound on the minimum positive probability of the underlying distribution. Hence, for $\mu \ll \log m$, one can adjust k to be something like m/μ. Then, we can leverage the results in [32] and work in the regime where every symbol in the extended sample (unseen) will appear at least μ times in expectation.

Citing the paper: https://arxiv.org/abs/1902.05616. Sure, we will cite the paper and comment on it appropriately.

The role of "$j \geq 1$" in Lemma 6. A typo, j should be s in the lemma. In fact, we removed this as it is unnecessary.