Author Feedback: Learning Some Popular Gaussian Graphical Models without Condition Number Bounds

We thank the reviewers for their careful feedback. Below we answer the questions raised by the reviewers:

1. Reviewer 1 suggested adding some more citations to further motivate the study of attractive GGMs, which we plan to add, and asked how our results relate to the recent paper of Wang, Roy, and Uhler [3]. In the paper [3], the authors studied the learning of attractive GGMs when they are promised to be well-conditioned (Condition 3.1 in their paper), which they note is the same situation studied in the previous analysis of CLIME and other methods. Since the goal of our work is to study efficient learnability without a condition number assumption, the results in this work and their work (where they were interested in adaptivity, i.e. minimizing the number of tuning parameters) are orthogonal, and ours remains the first to give condition-number independent results for learning attractive GGMs with a computationally efficient algorithm.
2. Review 1 also asked that we elaborate on why attractiveness is important from a practical point of view. In fact [3] and earlier works studying the MTP_2 property address this point. In phylogenetic applications, observed variables are often positively dependent because of shared ancestry [4]. In various copula models that are popular in finance we posit a latent global market variable that also leads to positive dependence [2]. Finally the Gaussian free field, which plays a central role in some areas of mathematical physics, is a special case. We will add this discussion to our paper since it is clearly an important point of the broader context. In fact attractiveness in the context of Gaussian graphical models has been studied for almost forty years since the work of Karlin and Rinott [1].
3. A couple reviewers noticed a typo in the algorithm description on page 4: when we are learning the neighborhood of node i, step 2 of the GREEDYANDPRUNE algorithm looks for j which minimizes $\widehat{\mathrm{Var}}(X_i|X_S, X_j)$ (the typo is we replaced i by j here). As also noted by a reviewer, this is correct in the more detailed algorithm description given in the Appendix.
4. Reviewer 1 asked if the variable t should enter into the loop in step 2 of GREEDYANDPRUNE. This is just a loop indexing variable because we want to run the greedy step T times (so at each step, S has size t).
5. There were some suggestions about what could be moved between the main body and the Supplementary Material, but the suggestions of different reviewers are currently in conflict (Reviewer 1 suggests more about attractive models and more technical content in the main body, Reviewer 4 suggests less of this and adding more of the experimental results). If the paper is accepted, hopefully there will be additional space to add some of this material.
6. Reviewer 2 asked about comparison of CPU runtimes. Though it is not provided in the main text of this version, it is in the supplementary (Table 2, which shows the sequential runtime is similar to other popular methods); we will add some mention of this to the main text. We note an advantage of this method over some alternatives like the graphical lasso is that it is "embarassingly parallelizable" (recovering the neighborhood of each vertex can be done in parallel).
7. The reviewers noticed a few other typos and suggested some other small edits for clarity — thanks, we plan to make these changes in the next version of this work.

References

[1] Samuel Karlin, Yosef Rinott, et al. Total positivity properties of absolute value multinormal variables with applications to confidence interval estimates and related probabilistic inequalities. *The Annals of Statistics*, 9(5):1035–1049, 1981.

[2] Alfred Müller and Marco Scarsini. Archimedean copulae and positive dependence. *Journal of Multivariate Analysis*, 93(2):434–445, 2005.

[3] Yuhao Wang, Uma Roy, and Caroline Uhler. Learning high-dimensional gaussian graphical models under total positivity without adjustment of tuning parameters. In *International Conference on Artificial Intelligence and Statistics*, pages 2698–2708, 2020.

[4] Piotr Zwiernik. *Semialgebraic statistics and latent tree models*. CRC Press, 2015.