- We thank the reviewers for the insightful reviews and valuable suggestions. We address the comments as follows.
- Reviewer #1:
- **Provide proof of Lemma 1:** The proof of Lemma 1 uses induction. We will add the proof to the supplementary.
- What is  $R_{\text{max}}$ , a maximum value of the reward function R? Yes, as defined in Section 2.1.
- Presentation issues of Algorithm 1: Many thanks! Yes, Algorithm 1 is meant for the synchronous case. We will
- clarify that lines 8 and 11 should be applied to all state action pairs. We will also specify the "converge" criterion in
- Algorithm 1, which can be, for example, when  $\sum_{i=t-N}^{t} \left\| Q_i^A Q_{i-1}^A \right\|_{\infty} < \varepsilon$  for a given  $N \ge 1$  and  $\varepsilon > 0$ .
- Discussion on the difficulties of analysis in the function approximation setting: This is a very important yet
- challenging problem. Even vanilla Q-learning with linear function approximation may not always converge and requires
- strong assumptions/conditions to converge, because the contraction property of the Bellman operator no longer holds in 10
- the function approximation setting. For double-O algorithm, it is likely that neither of the two parameters (corresponding 11
- to the two Q-functions) converges, and even if they both converge, it is unclear whether they converge to the same point. 12
- Characterizing the conditions for these two interconnected stochastic processes to converge can be difficult.
- Discussion on exploration policy in asynchronous version and Assumption 1: Yes, Assumption 1 is on a determin-
- istic exploration policy for the simplicity of presentation of the key insights. The reason for  $L \gg |S||A|$  is because 15
- in practice often some state-action pairs are repeatedly visited before the first visit of some other state-action pairs. 16
- L = |S||A| can rarely happen in practice due to stochastic visits of state-action pairs. Assumption 1 can be replaced 17
- by a high probability requirement. Our analysis can accommodate such a relaxation by additionally dealing with a 18
- conditional probability event. 19
- In Step I of Part I of proof of Theorem 1, provide intuition on the martingale difference sequence (MDS)  $z_t$  in
- the modeling of the interconnected stochastic processes: Indeed, interconnection of SAs introduces complication, as 21
- reflected by the non-MDS error sequence  $F_t$  (line 420 in supplementary). To handle the analysis of  $F_t$ , we decompose 22
- $F_t = F_t \mathbb{E}(F_t|\mathcal{F}_t) + \mathbb{E}(F_t|\mathcal{F}_t) := z_t + h_t$ , where we define  $z_t := F_t \mathbb{E}(F_t|\mathcal{F}_t)$  and  $h_t := \mathbb{E}(F_t|\mathcal{F}_t)$ . In this way,  $z_t$  is constructed to be an MDS by subtracting the conditional mean of  $F_t$  (a standard way to construct MDSs). The 23
- 24
- complication of non-MDS nature of  $F_t$  is captured by  $h_t$ , which we handle by exploiting a contraction-type property.
- Minor comments: Many thanks for the suggestions. We will make the changes in the revision. 26
- Reviewer #2:
- Discussion on double Q-learning resolving overestimation: Our analysis of the interconnected SAs can also provide
- some insights into how double Q-learning resolves overestimation. High-level speaking, the convergence of  $\|Q^A Q^*\|$  is obtained with high probability given the condition that  $\|Q^A Q^B\|$  can converge. This suggests that neither  $Q^A$  nor 29
- 30
- $Q^B$  can approach to  $Q^*$  alone too aggressively, which implies mitigation of overestimation. 31
- Numerical results: We have obtained some numerical results and will include them in the revision.
- **Intuition on**  $\hat{\tau}_1$ : We design the length of blocks in a recursive way, so that the ending time of the first block  $\hat{\tau}_1$
- determines the ending times of all following blocks. Then in our analysis,  $\hat{\tau}_1$  is determined to guarantee that the distance 34
- between the two Q-functions can be bounded blockwisely with high probability. 35
- Reviewer #3: 36
- **Lower bound on**  $\gamma$  (  $\gamma > 1/3$ ): In order for both  $\{G_k\}$  and  $\{D_k\}$  respectively serving as upper bounds on  $\|Q^A Q^B\|$  and  $\|Q^A Q^*\|$ , we construct  $G_k = \sigma D_k$  with  $\sigma = \frac{1-\gamma}{2\gamma}$  in Proposition 2 (which may not be the only design to serve the purpose). Then, since the convergence of  $\|Q^A Q^*\|$  is conditioned on the convergence of  $\|Q^A Q^B\|$ , we further
- 38
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- need  $D_k > G_k$ , which requires  $\gamma > 1/3$ . This may not be an essential requirement, but is rather a technical assumption
- due to the techniques we use.
- **About linear learning rate:** Great question! We are currently working on extending our techniques to the case with
- linear learning rate, and need to resolve some technical issues in order to obtain satisfactory results. 43
- Reviewer #4: Connection and difference with Even-Dar and Mansour (2003): The connection lies in that our work
- also used the blockwise analysis as in Even-Dar and Mansour (2003). The difference is that the analysis of double 45
- Q-learning requires to handle two interconnected SAs, and the analysis of single SA in Even-Dar and Mansour (2003) 46
- cannot be directly applied. Our technical novelty lies in designing the coupling relationship between two blockwise 47
- upper bounds and dealing with conditional bounds.
- **Definition between "synchronous" vs. "asynchronous":** Thanks for pointing this out. We will fix the statement of 49
- the synchronous algorithm in the revision.