- Thanks for your comments and helpful suggestions! We begin with responses that might interest multiple reviewers.
- 2 REVIEWER 4: Framework requires knowledge of P, but in many practical settings it is difficult/impossible to know P,
- 3 or control/design P.
- 4 We believe that the difficulty of knowing P in some practical settings makes our work *more* applicable, not less, in the
- 5 following sense. Suppose someone conducts a poll or survey of students and then makes a claim like "I estimate the
- 6 overall student body mean to be 74 based on the empirical mean of the respondents". No one truly knows the underlying
- 7 sampling distribution, P (different students have different response probabilities, there might be some independence,
- 8 but also likely correlations in participation between friends, etc.). Using our framework and algorithm, you could design
- 9 a variety of plausible sampling distributions, P, and then evaluate the worst-case expected error of the empirical mean
- with respect to these P's, which would provide compelling evidence for whether or not to believe the claim about the
- student body mean being 74. [Recall Prop 1 in Sec 2.1 that argues that we can evaluate the worst-case-expected-error of
- 12 a given/fixed (semi-linear) estimator such as the empirical mean, with respect to these "plausible" P's.]
- 13 In this sense, our framework and algorithm can be used to rigorously evaluate the stability of an estimator (in the above
- 14 example, the empirical mean) with respect to various "plausible" P's. We think this is a useful alternative to more
- standard approaches that evaluate the estimator with respect to other kinds of assumptions on the data. We plan to add a
- discussion of this to the paper and thank the reviewer for eliciting this alternate use case for our framework.
- 17 REVIEWER 2: Extension to ℓ_1 or ℓ_2 constraints instead of ℓ_{∞} .
- Good question. Our original motivation was for settings such as surveys/polling (e.g. COVID testing) where data values
- are binary or otherwise bounded in magnitude, and where ℓ_{∞} is the natural constraint. We agree that investigating
- 20 our model with respect to other constraints seems natural and worthwhile. We have done some preliminary work
- investigating the ℓ_2 case (not included in our submission): since the geometry of the ℓ_2 norm is so well behaved, there
- is potential for efficient algorithms that might even improve upon the $\pi/2$ approximation factor and surmount several of
- the more structural hurdles we encountered when investigating fully general (non semi-linear) estimators. We haven't
- 24 fleshed out any details, though we agree this is a very interesting direction.
- REVIEWER 4: Underlying domain is assumed to be finite (ie size n).
- 26 Our framework and definition of worst-case expected error apply, without modification, to infinite sets of underlying
- 27 datapoints. Additionally, it seems like an extension of our algorithm might be adapted to such settings. To briefly
- sketch this, suppose P corresponds to a joint distribution over an infinite (or continuous) domain indexing potential
- elements, and that each sample/target set in the support of P has size at most k. Very roughly, one can (1) draw many
- (i.e. poly(k)) sample/target sets from P, let Z denote the union of these sampled sets (together with the $\leq 2k$ elements
- of A, B—the actual sample/target sets) and then (2) let P' denote the restriction of P to those sample/target sets that
- have non-empty intersections with Z, and (3) run our algorithm on this (now finite) set Z and distribution P'. The one
- delicate step (that would take some space to fully describe) is that one must adjust P' to account for the possibility that
- This extension of our algorithm to infinite/continuously indexed data is certainly not trivial, and we haven't fleshed out

the measure in P that intersects Z in 1 point, might be (infinitely) larger than the measure intersecting in 2 points, etc.

- a formal proof of this. It does seem quite interesting—thanks for pointing out this direction.
- 37 REVIEWER 4: "Results mainly apply to scalar sets"

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- We mainly focus on scalar sets, as that seems like the natural starting point for explaining/exploring this framework.
- 39 Still, as our linear regression results (Thm 2) illustrate, the framework naturally extends to non-scalar settings, and one
- 40 can obtain interesting results in these non-scalar settings. We imagine that future work will likely explore a number of
- different non-scalar settings within the framework we propose.
- 42 REVIEWER 3: "try to compare this model with the common case where we have a distribution D on the population
- 43 and aim to minimize the statistic over D. Are there cases where a bound on the benchmark you propose implies a bound
- on the error of the estimator of expected value of the statistic over D?"
- We aren't sure we understand this question. Our interpretation of your question is the following: Suppose we have a
- distribution D (ie over all images) and the ultimate goal is to train a model to classify, say, cat images, that has small
- 47 expected loss wrt D. Given a model, how do we evaluate the expected loss over D? If we can sample from D, then
- great. If we can't sample from D, but instead can generate some (possibly dependent) set of samples S, drawn from a
- 49 joint distribution D' over sets of k = |S| samples, then we *could* use our framework to figure out how to estimate the
- expected loss over D based on the values of the loss on set S. [One natural example of such a D' might correspond to
- taking a sequence of k samples from a Markov Process whose stationary distribution is D...] We not sure if there are
- any 'standard' instances of such settings for which there are clean results to which we could compare with. [If this
- 53 doesn't address your question, please do clarify your question in your review...]