We thank all the reviewers for their time, feedback, and insightful comments.

2 Review #1.

- 3 β -approximation: By β -approximation, we mean an algorithm that approximates the optimum within a multiplicative
- factor of β . We will add a formal definition in the revision.
- 5 Comparison to [14]: This manuscript appeared on arXiv contemporaneously with our submission to NeurIPS.
- 6 Compared to our results, the main takeaways from [14] are: (i) their algorithms are heuristic and therefore do not
- 7 provably optimize for any objective such as value or revenue, and (ii) their algorithms are not provably fair under our
- 8 fairness constraints. Thus, unlike ours, their work does not seem to offer any formal guarantees.
- 9 For an experimental comparison, we tried the following two routes during the rebuttal process:
- 10 (i) We reached out to the authors for a pointer to their code. The authors responded that their code is currently unavailable as it is being revised based on the feedback they received.
- 12 (ii) We implemented their algorithms from scratch. During this process, we realized that replicating their results
- 13 is difficult due to the lack of a completely specified procedure for choosing the hyperparameters of their algorithm
- $(\alpha_0, \beta_0, \theta_1, \theta_2)$. For these reasons, to ensure a fair comparison with their work, we postpone the experimental evaluation
- of their algorithm pending the release of the new implementation by authors.

6 Review #2.

17 Thank you for the positive comments. We will address your suggestions and include the additional citation in the

18 revision.

- 19 **Review #3.**
- 20 Apologies for the typos and thank you for the careful reading.
- 21 Kleindessner et al. designed an algorithm for k-center with different type of fairness requirement. Instead of balancing
- different colors in each cluster, the goal is to pick centers (proportionally) from different colors. It is basically k-center
- 23 under partition matroid.
- 24 Thank you for articulating this clearly; we will add this in the revision.
- 25 Both the bounded representation and the representations from Bera et al. and Bercea et al. generalize the problem of
- 26 "balance". How do these different representations compare?
- 27 Bera et al. and Bercea et al. both introduce a generalized version of the constraint from [Ahmadian et al. 2019]. For
- each color, they specify the upper and lower bounds for their fractional representation in each cluster. Our results
- 29 continue to hold even for this general constraints, as long as we can find a fairlet decomposition satisfying the conditions
- of Theorems 8 and 10. We will add this remark to the revision.
- (Theorem 8): what is an upper bound on the value of m_f in terms of α (for general values of α)?
- For two colors with general $\alpha = r/(b+r)$, where $r \geq b$, we have $m_f \leq b+r$ [Chierichetti et al. 2019]. For multiple
- colors with $\alpha = 1/t$, we have $m_f \le 2t 1$. Note that the bound for multiple colors for general alpha is an open
- 4 question.
- Algorithm 1 step 2: how to find an initial solution?
- For two colors with general $\alpha=r/(b+r)$, we use the fairlet decomposition method proposed in [Chierichetti et
- 37 al. 2019]. For multiple colors with $\alpha = 1/t$, we use the method proposed in Lemma 24 in the Supplementary Material.
- In the revision, will also add further justifications for studying fairness in a hierarchical clustering setting.