A Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes] See Section 4.1
   (c) Did you discuss any potential negative societal impacts of your work? [No] This paper describes techniques that alleviate an existing flaw in applying privacy-preserving methods, which would improve their security in practice. Although there are potential negative impacts of privacy-preserving analysis, such as with fairness and the ability to study small populations, our paper does not exacerbate any already existing issues.
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes] See Assumption 3 and the setup for Theorem 2 and Corollary 1.
   (b) Did you include complete proofs of all theoretical results? [Yes] Sketches are in main document; complete proofs are in supplement.
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] Code is in the supplement.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] Code is in the supplement.
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [No] Figures 2 and 3 are simulations where the goal is precisely to characterize the marginal distributions of interest.
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [No] Figures 2 and 3 are primarily for illustrative purposes, and generating the figures does not require external computing resources.
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [Yes] Cited R and ggplot2
   (b) Did you mention the license of the assets? [Yes] All assets are under GNU GPL v3
   (c) Did you include any new assets either in the supplemental material or as a URL? [Yes] Code for the figures is in the supplement.
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A]
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A]
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]

B Proofs

B.1 Proof of Theorem 2

First note that if there are multiple $a \in \mathcal{Y}$ for which $L_X(a) = 0$, we can sample $a \sim \text{Unif}(\{a \in \mathcal{Y} \mid L_X(a) = 0\})$. Note that if $\mu_X(\{a \in \mathcal{Y} \mid L_X(a) = 0\}) > 0$, then we can directly apply the main result from [15]. Therefore we instead focus on the case where $\mu_X(\{a \in \mathcal{Y} \mid L_X(a) = 0\}) = 0$ in agreement with our assumption.

Next, the Metropolis-Hastings transition kernel is defined by:

$$\begin{aligned}\tilde{\Pi}_X(y, A) = \int_A \tilde{q}_X(y, y') \min\left\{1, \frac{\tilde{f}_X(y')}{\tilde{f}_X(y)}\right\} \, d\tilde{\nu}(y') + \\ \mathbb{1}_{\{y \in A\}} \left[1 - \int_{\mathcal{Y}} \tilde{q}_X(y, y') \min\left\{1, \frac{\tilde{f}_X(y')}{\tilde{f}_X(y)}\right\} \, d\tilde{\nu}(y')\right],\end{aligned}$$

Where:

$$\tilde{q}_X(y, y') \min\left\{1, \frac{\tilde{f}_X(y')}{\tilde{f}_X(y)}\right\} = \frac{1}{2}\left[\mathbb{1}_{\{y'=a\}} + q_X(y, y')\right] \min\left\{1, \frac{(1-k)f_X(y') + k\mathbb{1}_{\{y'=a\}}}{(1-k)f_X(y) + k\mathbb{1}_{\{y=a\}}}\right\}$$

The density $\tilde{f}_X$ is maximized at a by construction; note that this does not depend on the uniqueness of a. This implies:

$$\begin{aligned}\tilde{\Pi}_X(y, \{a\}) &\geq \int_{\{a\}} \tilde{q}_X(y, y') \min\left\{1, \frac{(1-k)f_X(y') + k}{(1-k)f_X(y) + k\mathbb{1}_{\{y=a\}}}\right\} \, d\tilde{\nu}(y') \\ &\geq \frac{k}{2} \triangleq \eta > 0\end{aligned}$$

Therefore the first condition is met, and we can use Algorithm 1 to perform perfect sampling. Let $N_{\text{Outer}} \sim \text{Geometric}(\eta_1)$ be the length of this outer loop.

Next, we need to characterize N_{Bern}, the number of Bernoulli factory flips necessary to sample from the Bernoulli distribution in Algorithm 1. Using the proposed algorithm in [14] and the minorization term above:

$$\mathbb{E}[N_{\text{Bern}}] \leq \frac{12}{\eta} = \frac{24}{k}$$

Next, we need to calculate, in the worst possible case, how many inner loop samples N_{Inner} are necessary to sample from the remainder in Algorithm 1:

$$\begin{aligned}\tilde{\Pi}_X(y, \mathcal{Y} \setminus \{a\}) = \int_{\mathcal{Y}\setminus\{a\}} \tilde{q}_X(y, y') \min\left\{1, \frac{\tilde{f}_X(y')}{\tilde{f}_X(y)}\right\} \, d\tilde{\nu}(y') + \\ \mathbb{1}_{\{y \in \mathcal{Y}\setminus\{a\}\}} \left[1 - \int_{\mathcal{Y}} \tilde{q}_X(y, y') \min\left\{1, \frac{\tilde{f}_X(y')}{\tilde{f}_X(y)}\right\} \, d\tilde{\nu}(y')\right] \\ \geq (1-k) \int_{\mathcal{Y}} q_X(y, y') \min\left\{1, \frac{f_X(y')}{f_X(y)}\right\} \, d\nu(y')\end{aligned}$$

Define:

$$p_{\text{Accept}}(y) \triangleq \int_{\mathcal{Y}} q_X(y, y') \min\left\{1, \frac{f_X(y')}{f_X(y)}\right\} \, d\nu(y')$$

Then:

$$\Pi_X(y, \mathcal{Y} \setminus \{a\}) \geq (1-k) \inf_{y \in \mathcal{Y}} p_{\text{Accept}}(y)$$

Finally, let $N_{\text{Nonatomic}}$ be the number of runs of Algorithm 1 to yield a perfect sample from the unmodified target distribution 2. Then $N_{\text{Nonatomic}} \sim \text{Geometric}(1-k)$. Combining all the results, the total number of proposed samples of all kinds N_{Total} is bounded above by:

$$\mathbb{E}[N_{\text{Total}}] \leq \mathbb{E}[N_{\text{Outer}}]\,\mathbb{E}[N_{\text{Bern}}]\,\mathbb{E}[N_{\text{Inner}}]\,\mathbb{E}[N_{\text{Nonatomic}}] \leq \frac{48}{k^2(1-k)^2 \inf_{y \in \mathcal{Y}} p_{\text{Accept}}(y)}$$

B.2 Proof of Corollary 1

The expected runtime bound follows immediately from the proof of Theorem 2 above. For the utility, recall that for the original exponential mechanism [17, Lemma 7]:

$$\mu_X(S_\varepsilon) \leq \frac{1}{\nu(S_{\varepsilon/2})} \exp\left(-\frac{\epsilon\varepsilon}{4\Delta_L}\right)$$

For the mechanism as implemented over the discrete atoms $\{y^{(l)}\}_{l=1}^{\ell}$, [8, Corollary 3.12] show that:

$$\mathbb{P}\left(\|L_X(Y)\| \geq \varepsilon \mid Y \in \{y^{(l)}\}_{l=1}^{\ell}\right) \leq \exp\left(-\frac{\epsilon}{2\Delta_L}\left(\varepsilon - \frac{2\Delta_L}{\epsilon}\log(\ell)\right)\right)$$

The utility result follows immediately from conditioning on the two mixture components.

B.3 Proof of Corollary 2

Using the same notation from the proof of the main theorem, the only modification necessary so that N_{prop} is 0-DP is that the data-dependent component, N_{inner} have a distribution independent of X. Using [3] Lemma 17, we can add geometric random noise to N_{Inner} for any iteration of the inner loop with probability depending on X. In particular, we assume an adversary knows a modified $\tilde{N}_{\text{Inner}}$ where:

$$\tilde{N}_{\text{Inner}} \triangleq N_{\text{Inner}}Z + (1-Z)(N_{\text{Inner}} + N_{\text{Wait}})$$

where:

$$Z \sim \text{Bernoulli}\left(\frac{\inf_{X\in\mathcal{X}^n}\inf_{y\in\mathcal{Y}} p_{\text{Accept}}(y)}{\inf_{y\in\mathcal{Y}} p_{\text{Accept}}(y)}\right), \quad N_{\text{Wait}} \sim \text{Geometric}\left(\inf_{X\in\mathcal{X}^n}\inf_{y\in\mathcal{Y}} p_{\text{Accept}}(y)\right)$$

Then:

$$\mathbb{E}\left[\tilde{N}_{\text{Inner}}\right] \leq \frac{2}{(1-k)\inf_{X\in\mathcal{X}^n}\inf_{y\in\mathcal{Y}} p_{\text{Accept}}(y)}$$

The corollary result then follows from replacing $\mathbb{E}\left[N_{\text{Inner}}\right]$ with $\mathbb{E}\left[\tilde{N}_{\text{Inner}}\right]$ in the the proof for Theorem 2.

B.4 Derivation for Example 1

$$f_X(y) = C_X \mathbb{1}_{\{y\in[0,1]^d\}} \exp\left(-\frac{\epsilon n}{2d}\left\|y - \overline{X}\right\|_1\right)$$

With integration constant:

$$C_X^{-1} \triangleq \int_{[0,1]^d} \exp\left(-\frac{\epsilon n}{2d}\left\|y - \overline{X}\right\|_1\right) d\nu(y)$$

Independent uniform proposal MCMC sampler:

$$\begin{aligned}
\inf_{X\in\mathcal{X}^n}\inf_{y\in\mathcal{Y}} \frac{q_X(y)}{f_X(y)} &= \left(\sup_{X\in\mathcal{X}^n}\sup_{y\in\mathcal{Y}} f_X(y)\right)^{-1} \\
&= \left(\inf_{X\in\mathcal{X}^n} C_X^{-1}\right)^{-1} \\
&= \prod_{j=1}^{d}\int_0^1 \exp\left(-\frac{\epsilon n}{2d}|y_j|\right) dy_j \\
&= \left(\frac{2d}{\epsilon n}(1 - e^{-\epsilon n/2d})\right)^d \triangleq \beta_{\text{MCMC,Unif}}
\end{aligned}$$

Laplace proposal MCMC sampler; first, let $Z \sim \text{MVLaplace}(x, \alpha)$ for $x \in [0,1]^d$ Then using the previous result:

$$\mathbb{P}(Z \in [0,1]^d) \geq \left(\frac{1}{\alpha}(1 - e^{-\alpha})\right)^d.$$

Then:

$$
\begin{aligned}
Q_X(y, y') &\geq Q_X(\overline{X}, y') \\
&\geq (2\alpha)^d \exp\left(-\left(\alpha d + \frac{\epsilon n}{2}\right)\right)\left(\frac{1}{\alpha}(1 - e^{-\alpha})\right)^d \triangleq \beta_{\text{MCMC,Lap}}
\end{aligned}
$$

Let $F(\cdot; b)$ be the CDF of the Laplace distribution with scale parameter b. Independent uniform proposal perfect sampler:

$$
\begin{aligned}
p_{\text{Accept}}(y) &= \int_{\mathcal{Y}} q_X(y, y') \min\left\{1, \frac{f_X(y')}{f_X(y)}\right\} \, d\nu(y') \\
&= \int_{[0,1]^d} \min\left\{1, \exp\left(-\frac{\epsilon n}{2d}\left(\left\|y' - \overline{X}\right\|_1 - \left\|y - \overline{X}\right\|_1\right)\right)\right\} \, d\nu(y') \\
&\geq \int_{[0,1]^d} \exp\left(-\frac{\epsilon n}{2d}\left(\left\|y' - \overline{X}\right\|_1\right)\right) \, d\nu(y') \\
&= \prod_{j=1}^{d} \int_0^1 \exp\left(-\frac{\epsilon n}{2d}|y'_j - \overline{X}_j|\right) \, dy'_j \\
&= \prod_{j=1}^{d} \frac{\epsilon n}{4d}\left(F\left(1 - \overline{X}_j; \frac{\epsilon n}{2d}\right) - F\left(-\overline{X}_j; \frac{\epsilon n}{2d}\right)\right)
\end{aligned}
$$

Laplace proposal perfect sampler:

$$
\begin{aligned}
p_{\text{Accept}}(y) &= \int_{\mathcal{Y}} q_X(y, y') \min\left\{1, \frac{f_X(y')}{f_X(y)}\right\} \, d\nu(y') \\
&= \int_{[0,1]^d} (2\alpha)^d \exp\left(-\alpha \left\|y - y'\right\|_1\right) \min\left\{1, \exp\left(-\frac{\epsilon n}{2d}\left(\left\|y' - \overline{X}\right\|_1 - \left\|y - \overline{X}\right\|_1\right)\right)\right\} \, d\nu(y') \\
&\geq \int_{[0,1]^d} (2\alpha)^d \exp\left(-\left(\frac{\epsilon n}{2d} + \alpha\right)\left(\left\|y' - \overline{X}\right\|_1\right)\right) \, d\nu(y') \\
&= (2\alpha)^d \prod_{j=1}^{d} \int_0^1 \exp\left(-\left(\frac{\epsilon n}{2d} + \alpha\right)|y'_j - \overline{X}_j|\right) \, dy'_j \\
&= (2\alpha)^d \prod_{j=1}^{d} \left(\frac{\epsilon n}{4d} + \frac{\alpha}{2}\right)\left(F\left(1 - \overline{X}_j; \frac{\epsilon n}{2d} + \alpha\right) - F\left(-\overline{X}_j; \frac{\epsilon n}{2d} + \alpha\right)\right)
\end{aligned}
$$

B.5 Simulation specification for Example 2

Constants:

$$
\begin{cases}
n \triangleq= 100 \\
p \triangleq 5 \\
\beta \triangleq (.1, .2, -.3, 0, 0)^T \\
\lambda \triangleq 1
\end{cases}
$$

Random variables:

$$
\begin{cases}
X_{ij} \sim \text{Beta}(5, 5) & i \in [n], j \in [p] \\
e_i \sim \text{Beta}(20, 20) & i \in [n] \\
Z_i \triangleq X_{i,\cdot}\beta + (2e_i - 1) & i \in [n]
\end{cases}
$$