A Proofs of Theorem 1 and Corollary 1

A.1 Proofs

Proof of Theorem 1. Recall the differential inequality (28) below from Proposition 2:

$$-\frac{d}{dt}W_2(p_t, q_t) \leq (L_f + L_s g^2)W_2(p_t, q_t) + g^2 b^{\frac{1}{2}}.$$

It can be solved by introducing the integrating factor,

$$I(t) := \exp\left(\int_0^t L_f(r) + L_s(r)g(r)^2 dr\right) \text{ where } b(t) := \mathbb{E}_{p_t}\left[\|\nabla \log p_t(x) - s_\theta(x,t)\|^2\right]. \quad (29)$$

As $\frac{d}{dt}I(t) = (L_f(t) + L_s(t)g(t)^2)I(t)$, the above inequality (28) can be written as

$$-\frac{d}{dt}\left\{I(t)W_2(p_t, q_t)\right\} \leq g(t)^2 I(t) b(t)^{\frac{1}{2}}.$$

Integrating both sides from 0 to T, we obtain that

$$I(0)W_2(p_0, q_0) - I(T)W_2(p_T, q_T) \leq \int_0^T g(t)^2 I(t) b(t)^{\frac{1}{2}} dt.$$

As $I(0) = 1$, we conclude that

$$W_2(p_0, q_0) \leq \int_0^T g(t)^2 I(t) b(t)^{\frac{1}{2}} dt + I(T)W_2(p_T, q_T). \quad (30)$$

□

Proof of Corollary 1. Let

$$J_I(\theta) = \int_0^T g(t)^2 I(t) b(t)^{\frac{1}{2}} dt. \quad (31)$$

Here, $I(t)$ and $b(t) = \mathbb{E}_{p_t}\left[\|\nabla \log p_t(x) - s_\theta(x,t)\|^2\right]$ are given in (29). The Cauchy-Schwarz inequality yields

$$J_I(\theta) \leq \left(\int_0^T 2g(t)^4 I(t)^2 \lambda(t)^{-1} dt\right)^{\frac{1}{2}} \left(\frac{1}{2}\int_0^T \lambda(t) b(t) dt\right)^{\frac{1}{2}}. \quad (32)$$

Since $\lambda = g^2$ and J_{SM} given in (5) satisfies

$$J_{SM}(\theta; \lambda) = \frac{1}{2}\int_0^T \lambda(t)\mathbb{E}_{p_t}\left[\|\nabla \log p_t(x) - s_\theta(x,t)\|_2^2\right] dt = \frac{1}{2}\int_0^T \lambda(t) b(t) dt, \quad (33)$$

we conclude (34):

$$W_2(p_0, q_0) \leq \sqrt{2\left(\int_0^T g(t)^2 I(t)^2 dt\right) J_{SM}} + I(T)W_2(p_T, q_T).$$

□

Remark 7. *It is expected that the parallel results hold for weak solutions to* (2) *and* (4) *by using suitable approximations. This approach has been studied for the Wasserstein contraction property in [8, Section 6.2].*

From (32) and (33) in the proof of Corollary 1, we obtain the following result for other choices of λ.

Corollary 4. *Let p_0 and q_0 be given in Theorem 1. Suppose that $g^4 I^2 \lambda^{-1}$ is integrable in* $[0, T]$. *Then the following inequality holds:*

$$W_2(p_0, q_0) \leq \sqrt{2\left(\int_0^T g(t)^4 I(t)^2 \lambda(t)^{-1} dt\right) J_{SM}} + I(T)W_2(p_T, q_T). \quad (34)$$

A.2 Technical lemmas

Lemma 1. *Let π_t be an optimal transport plan between p_t and q_t. Then, we have*

$$\mathbb{E}_{\pi_t}\left[(x-y)\cdot(v[q_t](y)-v[p_t](x))\right] \leq W_2(p_t,q_t)\left\{(L_f+L_s g^2)W_2(p_t,q_t)+g^2 b^{\frac{1}{2}}\right\} \tag{35}$$

where $b(t) := \mathbb{E}_{p_t}\left[\|\nabla \log p_t(x) - s_\theta(x,t)\|^2\right]$.

Proof. The left-hand side of (35) is given by

$$\begin{aligned}\mathbb{E}_{\pi_t}\left[(x-y)\cdot(v[q_t](y)-v[p_t](x))\right] &= \mathbb{E}_{\pi_t}\left[(x-y)\cdot(f(y,t)-f(x,t))\right]\\ &+ g^2\mathbb{E}_{\pi_t}\left[(x-y)\cdot(\nabla\log p_t(x)-s_\theta(y,t))\right]\\ &+\frac{g^2}{2}\mathbb{E}_{\pi_t}\left[(x-y)\cdot(\nabla\log q_t(y)-\nabla\log p_t(x))\right].\end{aligned}$$

In Lemma 2 below, we prove that the last term is less than or equal to zero.

In what follows, we estimate the first two terms. First, using the Lipschitzness of f in space, we get

$$\mathbb{E}_{\pi_t}\left[(x-y)\cdot(f(y,t)-f(x,t))\right] \leq L_f\mathbb{E}_{\pi_t}\left[\|x-y\|^2\right] = L_f W_2^2(p_t,q_t).$$

The last equality follows from the fact that π_t is an optimal plan between p_t and q_t.

Next, the second term $g^2\mathbb{E}_{\pi_t}\left[(x-y)\cdot(\nabla\log p_t(x)-s_\theta(y,t))\right]$ is the sum of the following two terms:

$$I_1 := g^2\mathbb{E}_{\pi_t}\left[(x-y)\cdot(s_\theta(x,t)-s_\theta(y,t))\right]$$

and

$$I_2 := g^2\mathbb{E}_{\pi_t}\left[(x-y)\cdot(\nabla\log p_t(x)-s_\theta(x,t))\right].$$

As shown above, the former one I_1 is bounded from above by $g^2 L_s W_2^2(p_t,q_t)$.

It suffices to find an upper bound on the latter one I_2. By the Cauchy-Schwarz inequality, we have

$$I_2 \leq g^2\mathbb{E}_{\pi_t}\left[\|x-y\|^2\right]^{\frac{1}{2}}\mathbb{E}_{\pi_t}\left[|\nabla\log p_t(x)-s_\theta(x,t)|^2\right]^{\frac{1}{2}}.$$

As the marginals of π_t are p_t and q_t, it holds that

$$\mathbb{E}_{\pi_t}\left[\|\nabla\log p_t(x)-s_\theta(x,t)\|^2\right] = \mathbb{E}_{p_t}\left[\|\nabla\log p_t(x)-s_\theta(x,t)\|^2\right].$$

As a consequence, we conclude that

$$I_1+I_2 \leq g(t)^2 W_2(p_t,q_t)\left\{L_s W_2(p_t,q_t)+b(t)^{\frac{1}{2}}\right\}$$

where $b(t) = \mathbb{E}_{p_t}\left[\|\nabla\log p_t(x)-s_\theta(x,t)\|^2\right]$. □

Before proving the lemma below, let us recall some basic definitions from the theory of optimal transport. The Wasserstein distance defined in (7) has an equivalent formulation:

$$W_2(\mu,\nu) = \inf\left\{\int_{\mathbb{R}^d}\|x-T(x)\|^2 d\mu : T_{\#}\mu = \nu\right\}^{\frac{1}{2}}. \tag{36}$$

The optimizer of the above problem is called the optimal map from μ to ν. It is well known that there exists a convex function ϕ such that $T = \nabla\phi$.

Lemma 2. $\mathbb{E}_{\pi_t}\left[(x-y)\cdot(\nabla\log q_t(y)-\nabla\log p_t(x))\right]$ *is nonpositive.*

Proof. Let T_t be an optimal transport map from p_t to q_t and a convex function ϕ_t satisfy $\nabla\phi_t = T_t$ for all $t\in[0,T]$. As in the proof of [7, Theorem 1], we have

$$\mathbb{E}_{\pi_t}\left[(x-y)\cdot(\nabla\log q_t(y)-\nabla\log p_t(x))\right] = -\mathbb{E}_{p_t}[\Delta\phi_t+\Delta\phi_t^*(\nabla\phi_t)-2d] \tag{37}$$

where ϕ_t^* is a convex conjugate of ϕ_t. The convexity of ϕ_t yields that $\Delta\phi_t+\Delta\phi_t^*(\nabla\phi_t)-2d$ and we conclude. □

B Further analysis of the upper bound

Proof of Corollary 2. Recall from Corollary 1 that

$$W_2(p_0, q_0) \le \sqrt{2\left(\int_0^T g(t)^2 I(t)^2 dt\right) J_{SM}} + I(T) W_2(p_T, q_T). \tag{38}$$

Based on the contraction property [8], we quantify the Wasserstein distance between p_T and ϕ.

$$W_2(p_T, \phi) \le \exp\left(-\int_0^T \frac{\beta(t)}{2} dt\right) W_2(p_0, \phi). \tag{39}$$

Using the above, the definition of $I(t)$, and $q_T = \phi$, we conclude (19). □

It worth noting that as a consequence of (39), $W_2(p_T, \phi)$ is small for an appropriate choice of T and $\beta(t)$.

B.1 Exponential convergence of h_t

For simplicity of notations, we define the norm in $L^2(\phi)$ as follows,

$$\|f\|_{L^2(\phi)} := \left(\int_{\mathbb{R}^d} f^2 d\phi\right)^{\frac{1}{2}}, \tag{40}$$

where ϕ is given in (18). In addition, assume that

$$\beta(t) > c > 0 \text{ for all } t \ge 0. \tag{41}$$

Lemma 3. *Under the same setting as in Corollary 2, we have*

$$\|h_t - 1\|_{L^2(\phi)} \le \exp\left(-\frac{\sigma^2 \lambda}{2} \int_0^T \beta(t) dt\right) \|h_0 - 1\|_{L^2(\phi)}. \tag{42}$$

where $h_t = p_t/\phi$ for some constant $\lambda > 0$. In particular, h_t exponentially converges to 1 *in $L^2(\phi)$ for β satisfying* (41).

For a constant function β, Lemma 3 is proven in [23]. For the sake of completeness, we provide the proof, which is a small modification of [23, Section 2].

Proof of Lemma 3. In our case, the equation (2) of p_t is given by

$$\partial_t p - \frac{\beta}{2}\left(\nabla \cdot (px) + \sigma^2 \Delta p\right) = 0, \;\; p(\cdot, 0) = p_0. \tag{43}$$

By direct computations, we obtain the equation of $h(x,t) = h_t(x)$ as follows:

$$\partial_t h - \frac{\beta}{2}\left(-\nabla h \cdot x + \sigma^2 \Delta h\right) = 0, \;\; h(\cdot, 0) = h_0. \tag{44}$$

We estimate $\|h - 1\|^2_{L^2(\phi)}$ by differentiating it with respect to time. Using the integration by parts and Poincaré inequality, we obtain that

$$\frac{d}{dt}\|h - 1\|^2_{L^2(\phi)} = -\sigma^2 \beta(t) \|\nabla h\|^2_{L^2(\phi)} \le -\sigma^2 \lambda \beta(t) \|h - 1\|^2_{L^2(\phi)} \tag{45}$$

This yields (42). Lastly, for β satisfying (41), $\|h_t - 1\|^2_{L^2(\phi)} \le \exp(-\sigma^2 \lambda c t)\|h_0 - 1\|^2_{L^2(\phi)}$. Thus, we conclude the exponential convergence of h to 1. □

Remark 8. *The convergence of h_t to* 1 *can be shown under more general assumption: $\beta > 0$ satisfying*

$$\lim_{T \to \infty} \int_0^T \beta(t) dt = \infty.$$

Further analysis is plausible based on the techniques in the study of partial differential equations.

Remark 9. *As p_t is given as the convolution between p_0 and the Gaussian distribution, it is smooth for $t > 0$. Therefore, h_t is also smooth, and the higher-order derivatives of h_t are all bounded. As a consequence, the above result combined with Gagliardo–Nirenberg interpolation inequality yield that the gradient of h, Dh, and the Hessian of h, D^2h, also converge to zero in $L^2(\phi)$.*

Remark 10. *Proving the uniform convergence of h, Dh, or D^2h requires an additional technical assumption that the support of h is bounded. Under the assumption, another interpolation inequality, Agmon's inequality, yields the desired uniform convergence result.*

B.2 Estimation of L_s

In this subsection, we investigate the estimation of L_s. If J_{SM} is sufficiently small, then s_θ is close to $\nabla \log p_t$. We first investigate the one-sided Lipschitz constant of $\nabla \log p_t$.

Lemma 4. *Under the same setting as in Corollary 2, $\nabla \log p_t$ satisfies the one-sided Lipschitz condition with a constant $(-\sigma^{-2} + \|D^2(\log h)\|_\infty)$ i.e.,*

$$(\nabla \log p_t(x) - \nabla \log p_t(y)) \cdot (x - y) \leq (-\sigma^{-2} + \|D^2(\log h)\|_\infty)\|x - y\|^2. \tag{46}$$

where $\|\cdot\|_\infty$ denotes the supremum of the matrix norm,

$$\|D^2(\log h)\|_\infty := \sup_{x\in\mathbb{R}^d} \|D^2(\log h(x))\| = \sup_{x,y\in\mathbb{R}^d, \|y\|\leq 1} \|D^2(\log h(x))y\|. \tag{47}$$

Proof. From the definition of h_t, we have

$$\log p_t(x) = \log h_t(x) + \log \phi(x) = \log h_t(x) - \frac{x^2}{2\sigma^2} - c \tag{48}$$

for some constant c. As a consequence,

$$(\nabla \log p_t(x) - \nabla \log p_t(y)) \cdot (x - y) = (\nabla \log h_t(x) - \nabla \log h_t(y)) \cdot (x - y) - \sigma^{-2}\|x - y\|^2. \tag{49}$$

To prove (46), it suffices to estimate $(\nabla \log h_t(x) - \nabla \log h_t(y)) \cdot (x - y)$. From the fundamental theorem of calculus, we have

$$(\nabla \log h_t(x) - \nabla \log h_t(y)) = \int_0^1 D^2(\log h_t)(sx + (1-s)y)ds \cdot (x - y). \tag{50}$$

Using $|z^\top D^2(\log h_t(w))z| \leq \|D^2(\log h)\|_\infty \|z\|^2$ for all $w, z \in \mathbb{R}^d$, we have

$$(\nabla \log h_t(x) - \nabla \log h_t(y)) \cdot (x - y) \leq \|D^2(\log h_t)\|_\infty \|x - y\|^2 \tag{51}$$

and conclude (46). $\square$

Remark 11. *Based on the similar relation as in (50), the difference between L_s and $-\sigma^{-2} + \|D^2(\log h)\|_\infty$ can be estimated. More precisely, we have $L_s(t) = (-\sigma^{-2} + \|D^2(\log h)\|_\infty) + \epsilon(t)$. Here, $\epsilon(t)$ depends on the difference between s_θ and $\nabla \log p_t$. Therefore, it is expected that the upper bound of $\int_0^T \epsilon(t)dt$ is given by J_{SM} under suitable regularity assumptions.*

C Proofs of Theorem 2 and Corollary 3

For a given t, let

$$J_{SM}(\theta, t) := \frac{1}{2}\, \mathbb{E}_{p_t(x)}[\|s_\theta(x, t) - \nabla_x \log p_t(x)\|^2], \tag{52}$$

and

$$J_{DSM}(\theta, t) := \frac{1}{2}\, \mathbb{E}_{p_0(x(0))p_{0t}(x|x(0))}[\|s_\theta(x, t) - \nabla_x \log p_{0t}(x|x(0))\|^2]. \tag{53}$$

Lemma 5. *(Appendix in [36])*

$$\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)] = \mathbb{E}_{p_0(x(0))p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))]. \tag{54}$$

Proof of Theorem 2. From Lemma 5 in [36], we have

$$\begin{aligned} J_{SM}(\theta,t) &= J_{DSM}(\theta,t) \\ &\quad + \frac{1}{2}(\mathbb{E}_{p_t(x)}\left[\|\nabla_x \log p_t(x)\|^2\right] - \mathbb{E}_{p_0(x(0))p_{0t}(x|x(0))}[\|\nabla_x \log p_{0t}(x|x(0))\|^2]) \end{aligned} \tag{55}$$

Note that

$$\begin{aligned} &\mathbb{E}_{p_t(x)}\left[\|\nabla_x \log p_t(x)\|^2\right] - \mathbb{E}_{p_0(x(0))p_{0t}(x|x(0))}[\|\nabla_x \log p_{0t}(x|x(0))\|^2] \\ &= \mathbb{E}_{p_t(x)}[(\nabla_x \log p_t(x))^\top(\nabla_x \log p_t(x))] \\ &\quad - \mathbb{E}_{p_0(x(0))}[\mathbb{E}_{p_{0t}(x|x(0))}[(\nabla_x \log p_{0t}(x|x(0)))^\top(\nabla_x \log p_{0t}(x|x(0)))|x(0)]] \\ &= \mathrm{Var}[(\nabla_x \log p_t(x))^\top] - \mathbb{E}[\mathrm{Var}[(\nabla_x \log p_{0t}(x|x(0)))^\top]|x(0)] \\ &\quad + (\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)])^\top(\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)]) \\ &\quad - \mathbb{E}_{p_0(x(0))}[(\mathbb{E}_{p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))|x(0)])^\top(\mathbb{E}_{p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))|x(0)])] \\ &\leq (\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)])^\top(\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)]) \\ &\quad - \mathbb{E}_{p_0(x(0))}[(\mathbb{E}_{p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))|x(0)])^\top(\mathbb{E}_{p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))|x(0)])] \end{aligned} \tag{56}$$

where the inequality comes from the law of total variance and our condition:

$$\begin{aligned} &\mathrm{Var}[(\nabla_x \log p_t(x))^\top] - \mathbb{E}[\mathrm{Var}[(\nabla_x \log p_{0t}(x|x(0)))^\top|x(0)]] \\ &= \mathrm{Var}[\mathbb{E}[(\nabla_x \log p_{0t}(x|x(0)))^\top|x(0)]] \\ &= 0. \end{aligned} \tag{57}$$

Then, we have

$$\begin{aligned} &(\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)])^\top(\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)]) \\ &\quad - \mathbb{E}_{p_0(x(0))}[(\mathbb{E}_{p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))|x(0)])^\top(\mathbb{E}_{p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))|x(0)])] \\ &\leq (\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)])^\top(\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)]) \\ &\quad - (\mathbb{E}_{p_0(x(0))}\mathbb{E}_{p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))|x(0)])^\top(\mathbb{E}_{p_0(x(0))}\mathbb{E}_{p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))|x(0)]) \\ &= (\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)] - \mathbb{E}_{p_0(x(0))}\mathbb{E}_{p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))|x(0)])^\top \\ &\quad (\mathbb{E}_{p_t(x)}[\nabla_x \log p_t(x)] + \mathbb{E}_{p_0(x(0))}\mathbb{E}_{p_{0t}(x|x(0))}[\nabla_x \log p_{0t}(x|x(0))|x(0)]) \\ &= 0, \end{aligned} \tag{58}$$

where first inequality comes from Jensen's inequality, and the last equality comes from eq. (54).

Recall that $J_{SM}(\theta,\lambda) = \int_0^T \lambda(t) J_{SM}(\theta,t)dt$ and $J_{DSM}(\theta,\lambda) = \int_0^T \lambda(t) J_{DSM}(\theta,t)dt$, $\lambda(t) > 0$, we have that $J_{DSM} \geq J_{SM}$. Plugging it in eq. (34), we can get eq. (22). $\square$

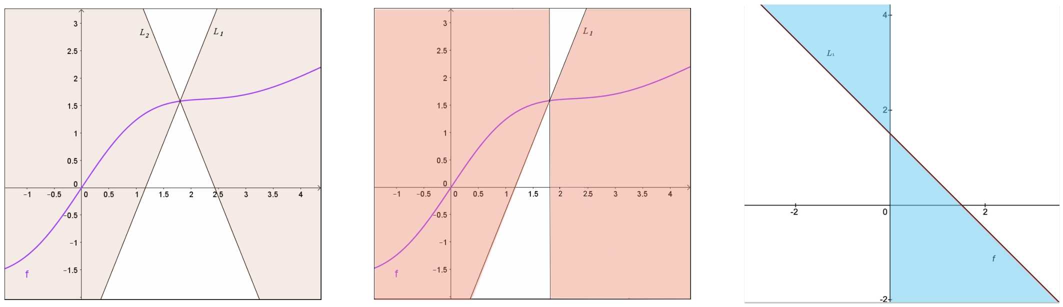
Proof of Corollary 3. We have $p_{0t}(x|x(0)) = \mathcal{N}(\sqrt{\bar{\alpha}_t}x(0), (1-\bar{\alpha}_t)I)$ where $\bar{\alpha}_t = \prod_{r=1}^t (1-\beta_t)$, which can be inferred from $p(x(t)|x(t-1)) = \mathcal{N}(\sqrt{1-\beta_t}x(t-1), \beta_t I)$.

Thus we can show that $\mathbb{E}[\nabla_x \log p_{0t}(x|x(0))^\top|x(0)]$ is constant with respect to $x(0)$: recall $p_{0t}(x|x(0)) = \mathcal{N}(\sqrt{\bar{\alpha}_t}x(0), (1-\bar{\alpha}_t)I)$, we have $\nabla_x \log p_{0t}(x|x(0)) = -((1-\bar{\alpha}_t)I)^{-1}(x(t) - \sqrt{\bar{\alpha}_t}x(0))$, which is a linear function of $x(t) - \sqrt{\bar{\alpha}_t}x(0)$. Using the Gaussian density function, we have:

$$\int p_{0t}(x|x(0))\nabla_x \log p_{0t}(x|x(0))dx = 0, \tag{59}$$

As a result, $\mathrm{Var}[\mathbb{E}[(\nabla_x \log p_{0t}(x|x(0)))^\top|x(0)]] = 0$, which satisfies the condition of eq. (21) in Theorem 2.

Note that here the assumption of f and g is only a sufficient condition for $\mathrm{Var}[\mathbb{E}[(\nabla_x \log p_{0t}(x|x(0)))^\top|x(0)]] = 0$. In fact, any conditional distribution p_{0t} that satisfies $\mathrm{Var}[\mathbb{E}[(\nabla_x \log p_{0t}(x|x(0)))^\top|x(0)]] = 0$ can lead to the same conclusion. $\square$



(a) Two-sided Lipschitz (b) One-sided Lipschitz; $L > 0$ (c) One-sided Lipschitz; $L < 0$

Figure 5: Two-sided and one-sided Lipschitzness.

D One-sided Lipschitzness

For an arbitrary Lipschitz function $F : \mathbb{R}^d \to \mathbb{R}^d$, the Cauchy-Schwarz inequality yields that

$$(F(x) - F(y)) \cdot (x - y) \leq \|F(x) - F(y)\| \|x - y\| \leq L\|x - y\|^2 \tag{60}$$

where L is the Lipschitz constant of F. Therefore, all Lipschitz function satisfies the one-sided Lipschitz condition:

$$(F(x) - F(y)) \cdot (x - y) \leq L\|x - y\|^2. \tag{61}$$

As pointed out earlier in Section 3.2, the one-sided Lipschitz constant is not necessarily to be positive. For instance, if $F(x) = -ax + b$ for $a > 0$ and $b \in \mathbb{R}^d$, then $-a < 0$ can be the one-sided Lipschitz constant of F while its Lipschitz constant is $a > 0$. Figure 5 visualizes this.

Note that two-sided Lipschitzness is a subset of one-sided Lipscthizness. See Figure 6 as an example.

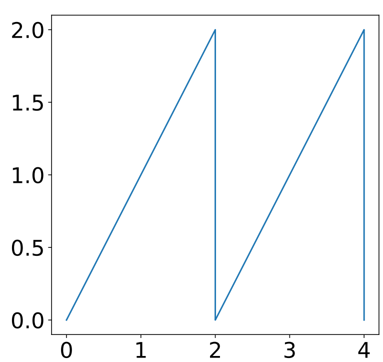
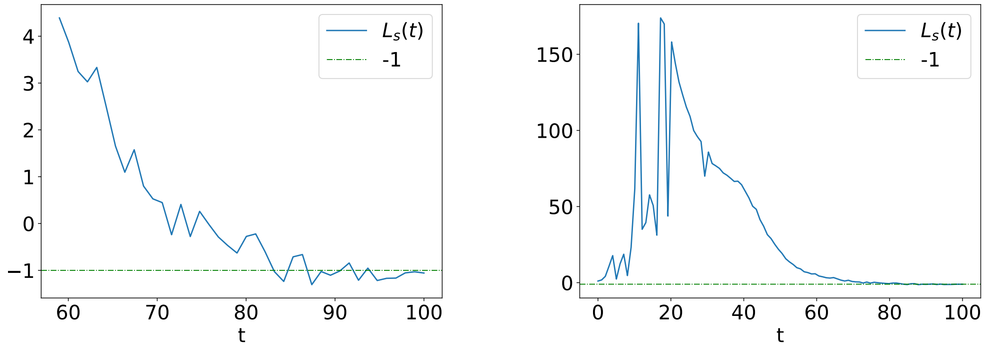


Figure 6: A function could be one-sided Lipschitz but not two-sided.

E Full plot of $L_s(t)$ when $T = 100$

See Fig. 7b.



(a) $L_s(t)$ when $t \geq 50$, $T = 100$.

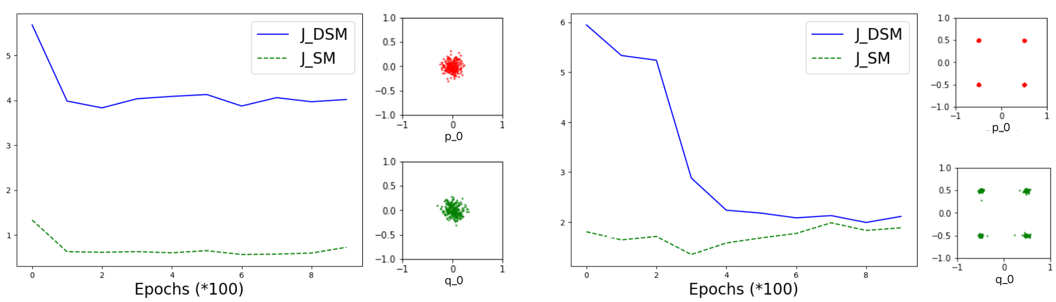
(b) Full plot of $L_s(t)$ when $T = 100$.

Figure 7: Plots of $L_s(t)$, $T = 100$.

F Numerical results on J_{DSM} upper-bounding J_{SM} in DDPM

To verify $J_{SM} \leq J_{DSM}$ in (22) for DDPM, we adopt the same datasets as in Fig 1, and the same training and evaluation settings in Section 4.1.

Moreover, to estimate J_{SM} numerically, we estimate $p_t(x)$ by performing Gaussian kernel density estimation with bandwidth = 0.05 on sampled data. $\nabla_x p_t(x)$ is estimated by central difference approximation with interval = 0.01. The resulting plots of J_{DSM} and J_{SM} are shown in Fig 8, which shows that J_{DSM} is an upper bound of J_{SM} in DDPM during training, where p_0 is the dataset, and q_0 is the generated data distribution at the convergence of training.



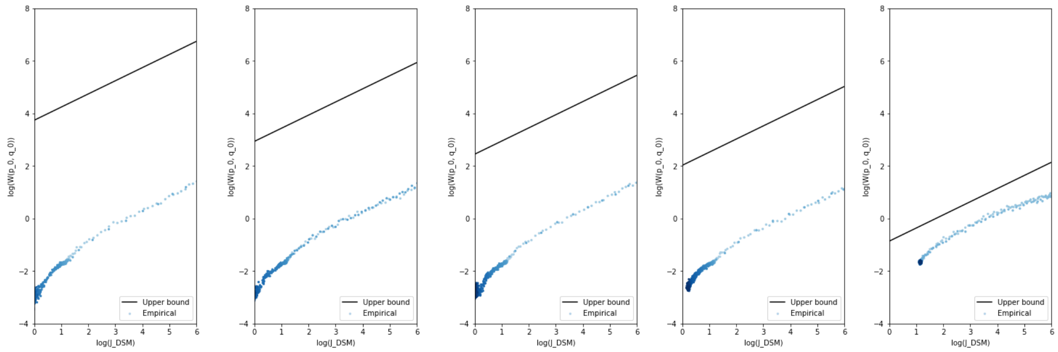
(a) 1 cluster in 2D

(b) 4 clusters in 2D

Figure 8: J_{SM} and J_{DSM} during training. The datasets are the same as 2D datasets in Fig 1. The training curves are obtained via training DDPM with modification of J_{DSM} loss.

G Log-log plots with weight decay

See Fig. 9.



(a) Coefficient=0.01 (b) Coefficient=0.1 (c) Coefficient=0.5 (d) Coefficient=1 (e) Coefficient=5

Figure 9: Log-log plots for different weight decay coefficients. As the weight decay coefficient increases, the theoretical upper bound is approaching the empirical one.