Mathematics

Query paper:
Title: W-algebras as coset vertex algebras
Abstract: We prove the long-standing conjecture on the coset construction of the minimal series principal W-algebras of ADE types in full generality. We do this by first establishing Feigin's conjecture on the coset realization of the universal principal W-algebras, which are not necessarily simple. As consequences, the unitarity of the "discrete series" of principal W-algebras is established, a second coset realization of rational and unitary W-algebras of type A and D are given and the rationality of Kazama–Suzuki coset vertex superalgebras is derived.

Candidate papers:

1. **Title:** Introduction to W-algebras and their representation theory
   Abstract: These are lecture notes from author's mini-course on W-algebras during Session 1: "Vertex algebras, W-algebras, and application" of INdAM Intensive research period "Perspectives in Lie Theory", at the Centro di Ricerca Matematica Ennio De Giorgi, Pisa, Italy. December 9, 2014–February 28, 2015.

2. **Title:** Representation theory of W-algebras
   Abstract: We study the representation theory of the W-algebra $W_k(\bar{g})$ associated with a simple Lie algebra $\bar{g}$ at level k. We show that the "−" reduction functor is exact and sends an irreducible module to zero or an irreducible module at any level $k \in C$. Moreover, we show that the character of each irreducible highest weight representation of $W_k(\bar{g})$ is completely determined by that of the corresponding irreducible highest weight representation of affine Lie algebra g of $\bar{g}$. As a consequence we complete (for the "−" reduction) the proof of the conjecture of E. Frenkel, V. Kac and M. Wakimoto on the existence and the construction of the modular invariant representations of W-algebras.

3. **Title:** Rationality of W-algebras: principal nilpotent cases
   Abstract: We prove the rationality of all the minimal series principal W-algebras discovered by Frenkel, Kac and Wakimoto, thereby giving a new family of rational and C2-cofinite vertex operator algebras. A key ingredient in our proof is the study of Zhu's algebra of simple W-algebras via the quantized Drinfeld-Sokolov reduction. We show that the functor of taking Zhu's algebra commutes with the reduction functor. Using this general fact we determine the maximal spectrums of the associated graded of Zhu's algebras of vertex operator algebras associated with admissible representations of affine Kac-Moody algebras as well.

4. **Title:** Unitary vertex operator algebras
   Abstract: Unitary vertex operator algebras are introduced and studied. It is proved that most well-known rational vertex operator algebras are unitary. The classification of unitary vertex operator algebras with central charge $c \leqslant 1$ is also discussed.

5. **Title:** Semi-infinite homological algebra
   Abstract: The paper provides a homological algebraic foundation for semi-infinite cohomology. It is proved that semi-infinite cohomology of infinite dimensional Lie algebras is a two-sided derived functor of a functor that is intermediate between the functors of invariants and coinvariants. The theory of two-sided derived functors is developed. A family of modules including a module generalizing the universal enveloping algebra appropriate to the setting of two sided derived functors is introduced. A vanishing theorem for such modules is proved.

6. **Title:** The combinatorics of category O over symmetrizable Kac-Moody algebras
   Abstract: We show that the structure of a block outside the critical hyperplanes of category O over a symmetrizable Kac-Moody algebra depends only on the corresponding integral Weyl group and its action on the parameters of the Verma modules. This is done by giving a combinatorial description of the projective objects in the block. As an application, we derive the Kazhdan-Lusztig conjecture for nonintegral blocks from the integral case for finite or affine Weyl groups. We also prove the uniqueness of Verma embeddings outside the critical hyperplanes.

Exemplary analysis:

1. **Relevance:** This paper likely provides foundational knowledge on W-algebras, which is essential for understanding the broader context of the query paper's research.
   Reason for Citation: It might have been cited to give readers an introductory overview of W-algebras and their representation theory, setting the stage for the more specific discussions that follow in the query paper.
2. **Relevance:** The query paper's focus on the coset construction and realization of W-algebras directly relates to their representation theory.
   Reason for Citation: This cited paper, which delves into the representation theory of W-algebras and proves certain conjectures related to modular invariant representations, could provide theoretical underpinnings or methodologies that the query paper builds upon or contrasts with its findings.
3. **Relevance:** Rationality is a key concept in the study of vertex algebras, including W-algebras.
   Reason for Citation: The query paper's mention of establishing the unitarity of the "discrete series" of principal W-algebras and the rationality of Kazama–Suzuki coset vertex superalgebras suggests a direct link to the discussions of rationality in this cited paper. The methods or results regarding the rationality of minimal series principal W-algebras could be foundational to the query paper's arguments or findings.
4. **Relevance:** It likely provides definitions, criteria, or examples of unitarity in vertex operator algebras that the query paper refers to or builds upon in its own discussions of unitarity.
   Reason for Citation: Given that the query paper establishes the unitarity of certain W-algebras, this cited paper's focus on unitary vertex operator algebras is highly relevant.
5. **Relevance:** The query paper might delve into complex algebraic structures and constructions in proving its conjectures, where semi-infinite homological algebra could play a crucial role.
   Reason for Citation: This cited paper's development of the theory of two-sided derived functors and its application to infinite dimensional Lie algebras might provide necessary theoretical tools or frameworks that the query paper utilizes.

6. **Relevance:** This paper's exploration of the structure of category O over symmetrizable Kac-Moody algebras and its implications for the Kazhdan-Lusztig conjecture could be relevant to the query paper's discussions on the coset construction and realization of W-algebras.

   Reason for Citation: The combinatorial and structural insights from this cited paper might inform or support the query paper's theoretical or methodological approaches.

Exemplary ranking: Ranked order: paper 2, paper 3, paper 4, paper 5, paper 6, paper 1

1. **Explanation:** Although this paper likely provides a valuable introductory overview of W-algebras, for a follow-up study that aims to build directly on the specific findings and discussions of the query paper, it might be considered foundational but not as immediately critical as the other, more specialized papers.

2. **Explanation:** This paper is likely the most directly relevant to a follow-up study, given its focus on the representation theory of W-algebras, which is central to understanding the coset construction and realization discussed in the query paper. A follow-up study would benefit from the theoretical underpinnings and methodologies related to the representation theory of W-algebras.

3. **Explanation:** Since the query paper deals with the rationality of Kazama–Suzuki coset vertex superalgebras and establishes the unitarity of the "discrete series" of principal W-algebras, a follow-up study focusing on these aspects would find this paper crucial for understanding the criteria and implications of rationality in W-algebras.

4. **Explanation:** For research that aims to delve deeper into the unitarity of W-algebras, this paper would provide essential background and criteria for unitarity in vertex operator algebras, which is necessary for extending the discussions on unitarity presented in the query paper.

5. **Explanation:** Given the complex algebraic structures and constructions involved in the coset construction of W-algebras, a paper focusing on semi-infinite homological algebra would be instrumental for a follow-up study looking to explore or expand upon the algebraic frameworks utilized in the query paper.

6. **Explanation:** While this paper might offer valuable insights into the combinatorial and structural aspects of algebraic theories relevant to W-algebras, it may be slightly less directly connected to the specific focus of the query paper compared to the others. However, it would still be relevant for a comprehensive understanding of the algebraic context.